УДК 519.004.942
DOI: 10.20535/SRIT.2308-8893.2018.3.02

ANALYSIS OF MOON'S GRAVITATIONAL-WAVE AND EARTH'S GLOBAL TEMPERATURE: INFLUENCE OF TIME-TREND AND CYCLIC CHANGE OF DISTANCE FROM MOON

YOSHIO MATSUKI, PETRO I. BIDYUK

Abstract. This research examined the influence of Moon's gravitational-wave to Earth's global warming process and the effects of time-trend and cyclic change of the distance between Moon and Earth. In the pervious research [1], we found that the Moon's gravitational-wave could influence the process of the Earth's global warming; and, we also found that Moon's cyclic movement around Earth needed to be further investigated, because it gave a unique pattern of distribution in the data for the empirical analysis; while both global temperature and global carbon-dioxide increase almost linearly in the time-series. In this research we added dummy binary variables that simulate the trend of time and the cyclic changes. As a result we confirmed that the influence of Moon's gravitational-wave is significant in the process of rising global temperature on Earth.

Keywords: global temperature, Moon's gravitational-wave, trend removal, cyclic change.

INTRODUCTION

Our previous research [1] investigated the influence of Moon's gravitational-wave to the process of Earth's global warming with the methodology of empirical analysis with the database of Earth's global temperature and global carbon dioxide as well as the distance between Moon and Earth. Then, the result of the analysis suggested that there was a possibility such that Moon's gravitational-wave influenced Earth's atmospheric temperature than global carbon dioxide could do. However, the uncertainty of the analysis [1] was also large, due to the cyclic change of the distance between Moon and Earth. In the previous research [1], we attempted to reduce this uncertainty, by assuming pure-heteroskedasticity and the first-order autoregressive process of Generalized Classical Regression models; however, we didn't know if these assumptions were appropriate in order to explain the cyclic change of the distance between Moon and Earth.

Considering the above result [1], in this research, we continued the empirical analysis of the same database with different techniques: maximum-likelihood estimation, trend removal, and removal of the influence of the cyclic change of the distance between Moon and Earth, by adding binary variables.

The gravitational-wave was a theoretical possibility when we made the previous research [1]; also, we didn't calculate the intensity of the gravitational-wave. Instead, we used the inverse of the squared distance between Moon and Earth as the surrogate of the gravitational-wave, because our mathematical method uses the deviations of the values of the variables, not necessarily the intensities of physical energy.

METHOD

The descriptive statistics of the data, from 1987 till 2009, of the global temperature (increased degree Celsius since 1978) [2], the global carbon dioxide (million tons) [3], the distance between Moon and Earth (r : kilometers) [4], and calculated $\frac{1}{r^2}$ ((kilometers)$^{-2}$), are shown in Table 1.

Table 1. Descriptive statistics

Variable	**Global Temperature °C ***	**CO_2 mil. tons****	**Distance between Moon and Earth *r*, km**	**$\frac{1}{r^2}$, km^2**
Mean	0,29130	$1{,}25165 \cdot 10^3$	$3{,}62618 \cdot 10^5$	$7{,}60509 \cdot 10^{-12}$
Standard deviation	0,12125	$2{,}14245 \cdot 10^2$	$5{,}98411 \cdot 10^2$	$2{,}51097 \cdot 10^{-14}$
Minimum	0,10000	$8{,}92000 \cdot 10^2$	$3{,}61583 \cdot 10^5$	$7{,}56999 \cdot 10^{-12}$
Maximum	0,43000	$1{,}62600 \cdot 10^3$	$3{,}63483 \cdot 10^5$	$7{,}64865 \cdot 10^{-12}$
Skewness	-0,21063	0,14292	-0,15249	0,15787
Kurtosis	1,29401	1,82491	1,67498	1,67879
Valid number of observations	23	23	23	23

* Increased degree Celsius since 1978.
** To convert these estimates to units of carbon dioxide (CO_2), simply multiply these estimates by 3,667 [3].

Analysis is made on the global temperature, the global CO_2 and $\frac{1}{r^2}$, with the following methods:

1. Maximum Likelihood Estimation. This method is an alternative approach, beside the Least Squares Estimation of Linear Classical Regression Model. The global temperature $Y=\{y_1,\ldots,y_n\}$, the constant value 1 (x_1), the measured global CO_2 (x_2), and $\frac{1}{r^2}$ (x_3), are transformed into the forms of $n\times 1$ vectors, y, x_1, x_2, x_3, where n is the number of observation, 23. Then $n\times k$ matrix $X=\{x_1,x_2,x_3\}$ is defined, where $k=\text{rank}(X)=3$ and X is non-stochastic. And, we assume that the data in Table 1 are samples from a real nature, which are multivariate normally distributed i.e. $Y\sim N(X\beta,\sigma^2 I)$, where $X\beta=\mu=E(Y)$, $\sigma^2 I=\Sigma=V(Y)$, I is a unit matrix whose diagonal elements are 1, and non-diagonal elements are 0, and $E(Y)$ is a mean value of Y ($\sigma_{ii}=\sigma^2$ for all i, and that $\sigma_{hi}=0$ for all $h\neq i$). And $f(Y)=(2\pi)^{-\frac{n}{2}}|\Sigma|^{-\frac{1}{2}}\exp\left(-\frac{w}{2}\right)$, where $w=\varepsilon'\Sigma^{-1}\varepsilon$, $\varepsilon=Y-\mu$, $|\Sigma|^{-\frac{1}{2}}=\frac{1}{\sqrt{\det(\Sigma)}}$, and in this model, $\Sigma^{-1}=\left(\frac{1}{\sigma^2}\right)I$, $|\Sigma|=(\sigma^2)^n$, $\varepsilon=Y-X\beta$.

And then $f(Y) = (2\pi)^{-\frac{n}{2}}(\sigma^2)^{-\frac{n}{2}} \exp\left[\frac{-\varepsilon'\varepsilon}{2\sigma^2}\right]$. Now, the Maximum Likelihood estimates of β and σ^2 are the values that maximize $\log L = -\left(\frac{n}{2}\right)\log(2\pi) - -\left(\frac{n}{2}\right)\log(\sigma^2) - \left(\frac{1}{2}\right)\frac{\varepsilon'\varepsilon}{\sigma^2}$. Then L is maximized by minimizing $\varepsilon'\varepsilon$ with respect to β. So, β is identical to the coefficients of the Least Squares Estimation of Linear Classical Regression Model ([1]). Now, inserting solution value for β makes $\varepsilon'\varepsilon = e'e$, with $e = Y - Xb$, which leaves the "concentrated log-likelihood function", as $L^*(\sigma^2) = L(b, \sigma^2) = -\left(\frac{n}{2}\right)\log(2\pi) - \left(\frac{n}{2}\right)\log(\sigma^2) - \left(\frac{1}{2}\right)\frac{e'e}{\sigma^2}$, to be maximized with respect to σ^2. The first derivative is $\frac{\partial L^*}{\partial \sigma^2} = -\frac{(n/2)}{\sigma^2} + \left(\frac{1}{2}\right)\frac{e'e}{\sigma^4}$. Equating $\frac{\partial L^*}{\partial \sigma^2}$ to zero and solving it gives the Maximum Likelihood estimator of σ^2 as $\frac{e'e}{n}$.

2. Trend Removal. At first, we define $x_1 = 1$ and $x_2 = t$, where t is a series of time. (Here we simply use a series of the values from 1 to 23 as the values of t). Then $X_1 = \{x_1, x_2\}$ and $X_2 = \{x_3, x_4\}$, where x_3 is the measured global CO_2, and x_4 is the $\frac{1}{r^2}$. And then, we calculate the residuals X_2^* from the regression of X_2 on X_1, following the matrix algebra bellow:

$Q_1 = X_1'X_1$, where X_1' is a transposed matrix of the matrix X_1;

$b = Q_1^{-1}X_1'X_2$, where Q_1^{-1} is an inversed matrix of the matrix Q_1;

$\hat{X}_2 = X_2 b$;

$X_2^* = X_2 - \hat{X}_2$.

Now X_2^* is the de-trended values of $X_2 = \{x_3, x_4\}$. We also calculate the de-trended values of Y (global temperature), by calculating $\hat{Y} = Yb$, and then $Y^* = Y - \hat{Y}$, where Y^* is the de-trended values of Y.

And then, we implement the Least Squares Estimation of Linear Classical Regression Model of the de-trended global temperature Y^* over X_2^*, with the following steps:

$Q^* = X_2^{*}{}'X_2^*$;

$b_1 = Q^{*-1}X_2^{*}{}'Y^*$;

$\hat{Y}^* = Y^* b_1$: expected de-trended global temperature Y;

$e = Y^* - \hat{Y}^*$;

$V(b_1) = \dfrac{e'e}{n-k} Q_1^{*-1}$.

And square-root of the diagonal elements of $V(b_1)$ are the standard errors of elements of the estimated coefficient-vector, b_1.

3. Removal of Seasonal (cyclic) Influence. Moon and Earth became closer every 4 years as shown in Fig. 1. In order to remove (de-seasonalize) the influence of the cyclic pattern from the explanatory variables (the measured global CO_2 and $\frac{1}{r^2}$), at first, we define four binary dummy variables:

$$x_1 = \begin{Bmatrix} 1 \text{ in } 1987, 1991, 1995, 1999, 2003, 2007 \\ 0 \text{ otherwise} \end{Bmatrix};$$

$$x_2 = \begin{Bmatrix} 1 \text{ in } 1988, 1992, 1996, 2000, 2004, 2008 \\ 0 \text{ otherwise} \end{Bmatrix};$$

$$x_3 = \begin{Bmatrix} 1 \text{ in } 1989, 1993, 1997, 2001, 2005, 2009 \\ 0 \text{ otherwise} \end{Bmatrix};$$

$$x_4 = \begin{Bmatrix} 1 \text{ in } 1990, 1994, 1998, 2002, 2006 \\ 0 \text{ otherwise} \end{Bmatrix}.$$

Then we set $X_1 = \{x_1, x_2, x_3, x_4\}$. Then we calculate the Least Squares Estimation of Linear Classical Regression Model of the global temperature Y over X_1 in order to get residuals $Y^* = Y - \hat{Y}$, with the following steps:

$Q = X_1{}' X_1$;

$b_1^* = Q^{-1} X_1{}' Y$;

$\hat{Y} = X_1 b_1^*$: expected global temperature Y ;

$Y^* = Y - \hat{Y}$.

Now the elements of Y^* are the de-seasonalized values of global temperature.

And then we remove the influence of the cyclic change of the distance between Moon and Earth from the measured global CO_2 (z_2), and $\frac{1}{r^2}$ (z_3). For this purpose, at first, we regress $X_4 = \{z_2, z_3\}$ on the dummy variables X_1, to get the coefficient $F = (X_1{}' X_1)^{-1} X_1 X_4$ and residuals $X_4^* = X_4 - X_1 F$, which removes the influence of the cyclic change of the distance between Moon and Earth from X_4. Then we make the Least Squares Estimation of the de-seasonalized variable Y^* on the de-seasonalized explanatory variables X_4^*, from which the influence of cyclic change of the distance between Moon and Earth has been removed, with the following steps:

$Q_5 = X_4^{*}{}' X_4^{*}$;

$b_5 = Q_5^{-1} X_4^{*}{}' Y^{*}$;

$e_5 = Y^{*} - X_4^{*} b_5$;

$V(b_5) = \frac{e_5' e_5}{n-k} Q_5^{-1}$.

And square-root of each diagonal element of $V(b_5)$ is the standard error of each element of the estimated coefficient-vector b_5.

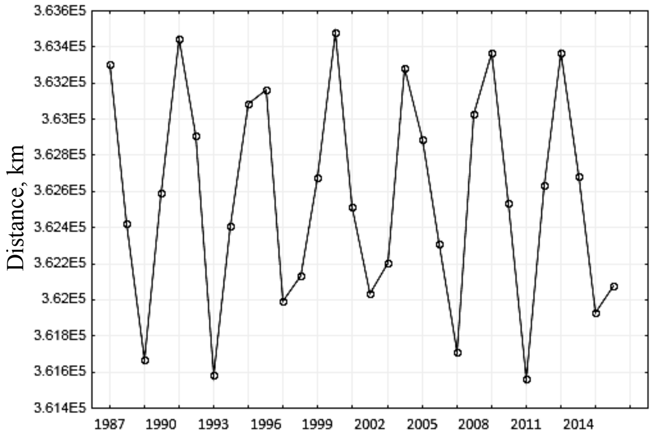

Fig. 1. Distance between Moon and Earth [1]

RESULTS

1. Result of Maximum Likelihood Estimation. Table 2 and Table 3 show the result of the Maximum Likelihood Estimation.

Table 2. Maximum Likelihood Estimation: coefficients and standard errors

Variable	Coefficient	Standard error*
Intercept (1)	-1,17897	$1{,}67861 \cdot 10^{3}$
CO_2	$5{,}32537 \cdot 10^{-4}$	$2{,}95105 \cdot 10^{-2}$
$\frac{1}{r^2}$	$1{,}05675 \cdot 10^{11}$	$2{,}19071 \cdot 10^{14}$

*Each standard error of each coefficient is square-root of diagonal element in Table 3.

Table 3. Variances and Covariances of Maximum Likelihood Estimation

Variable	Intercept	CO_2	$\frac{1}{r^2}$
Intercept (1)	$2{,}81774 \cdot 10^{6}$	-16,03004	$-3{,}67640 \cdot 10^{17}$
CO_2	-16,03004	$8{,}70867 \cdot 10^{-4}$	$1{,}95292 \cdot 10^{12}$
$\frac{1}{r^2}$	$-3{,}67640 \cdot 10^{17}$	$1{,}95292 \cdot 10^{12}$	$4{,}79922 \cdot 10^{28}$

2. Result of Trend Removal. At first, we made the regression of X_2 on X_1. The calculated coefficient b is shown in Table 4.

Table 4. Computing b: $b = Q_1^{-1} X_1' X_2$

$8{,}75075 \cdot 10^{2}$	$7{,}60732 \cdot 10^{-12}$
31,38142	$-1{,}85867 \cdot 10^{-16}$

And then we calculated the de-trended values of $X_2 = \{x_3, x_4\}$, where x_3 is CO_2 and x_4 is $1/r^2$, by calculating $\hat{X}_2 = X_2 b$, and then $X_2^* = X_2 - \hat{X}_2$, where X_2^* is the de-trended values of X_2. We also calculated the de-trended values of Y (global temperature), by calculating $\hat{Y} = Yb$, and then $Y^* = Y - \hat{Y}$, where Y^* is the de-trended values of Y. The descriptive statistics of the adjusted (de-trended) values (Y^* and X_2^*) are shown in Table 5. The global temperatures before and after the removal of trend are shown in Fig. 2, and the values of CO_2 and $1/r^2$ before and after the removal of trend are shown in Fig. 3.

Table 5. Descriptive statistics of de-trended values of global temperature, CO_2 and $1/r^2$

Variable	**Global Temperature** °C	**CO_2, mil. tons**	$1/r^2$, **km**^{2}
Mean	$-1{,}48809 \cdot 10^{-9}$	$-7{,}25622 \cdot 10^{-8}$	$-9{,}10100 \cdot 10^{-21}$
Standard deviation	$3{,}07795 \cdot 10^{-2}$	24,50602	$2{,}50781 \cdot 10^{-14}$
Minimum	$-6{,}15217 \cdot 10^{-2}$	-33,79644	$-3{,}59113 \cdot 10^{-14}$
Maximum	$4{,}22332 \cdot 10^{-2}$	60,53360	$4{,}26366 \cdot 10^{-14}$
Skewness	-0,28508	0,41789	0,14935
Kurtosis	1,89185	2,41821	1,66733
Valid number of observations	23	23	23

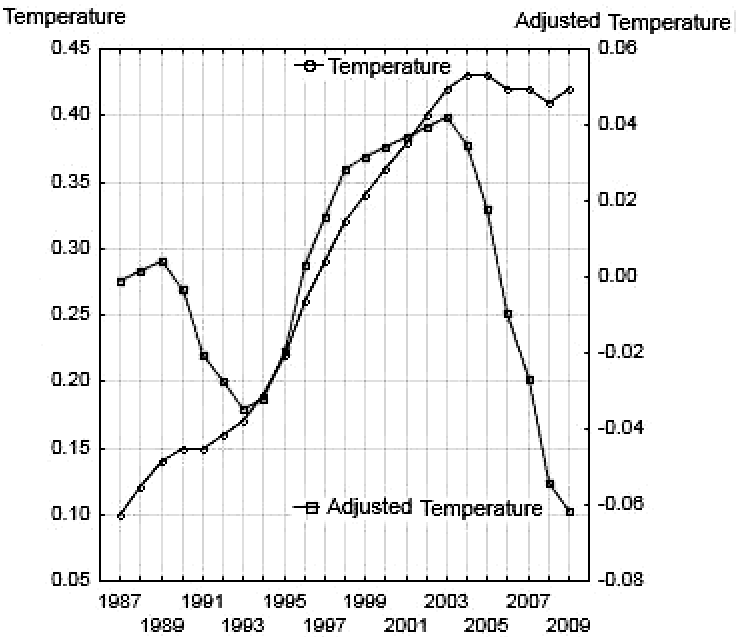

Fig. 2. Global temperatures with and without trend removal (Y^* and Y)

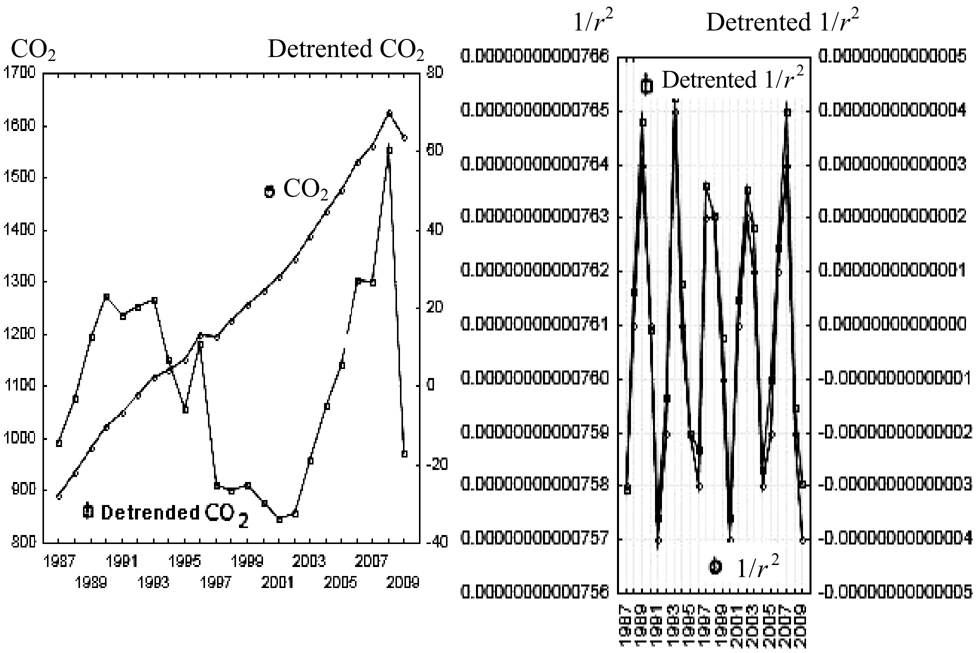

Fig. 3. Comprting CO_2 and $1/r^2$ before and after trend removal (X_4 and X_4^*)

And then we calculated the Least Squares Estimation of the de-trended global temperature Y^* over X_2^*. At first, we calculated $Q^* = X_2^{*}{}' X_2^*$, and $b_1 = Q^{*-1} X_2'^* Y^*$. Table 6 shows the calculated b_1.

Table 6. Comprting b_1: $b_1 = Q^{*-1} X_2'^* Y^*$

Variable	**Values of b_1**
Intercept* for $x_1 = 1$	b_1' : 7,94412 · 10^{-2}
Intercept* for $x_2 = t$	b_1' : 1,76544 · 10^{-2}
CO_2	-1,15220 · 10^{-5}
$1/r^2$	1,89763 · 10^{9}

*To calculate b_1' for the intercepts (each of x_1 and x_2), we calculated $b^* = Q_1^{-1} X_1' Y$, and then $F = Q_1^{-1} X_1' X_2$, and then $b_1' = b^* - F b_1$.

And then we calculated $e = Y^* - \hat{Y}^*$ and $V(b_1) = \frac{e'e}{n-k} Q_1^{*-1}$ to calculate the standard errors of elements of the estimated coefficient-vector b_1. Table 7 shows the calculated values of $V(b_1)$, and Table 8 shows the calculated values of standard errors of b_1.

Table 7. Comprting V: $V(b_1)$

Variable	**CO_2**	$1/r^2$
CO_2	1,02316 · 10^{-9}	-1,68386 · 10^{5}
$1/r^2$	-1,68386 · 10^{5}	2,84905 · 10^{19}

Table 8. Standard errors of b_1

Variable	Standard errors of b_1
Intercept* for $x_1 = 1$	$1{,}39138 \cdot 10^{-2}$
Intercept* for $x_2 = t$	$1{,}01477 \cdot 10^{-3}$
CO_2	$3{,}19869 \cdot 10^{-5}$
$1/r^2$	$5{,}33765 \cdot 10^{9}$

*To calculate standard errors for the intercepts (x_1 and x_2), we calculated $N_1 = X_1 Q_1^{-1} X_1'$ and $e_1 = (I_1 - N_1)Y^*$, $V(b_1) = \frac{e_1' e_1}{n-k} Q_1^{-1}$, and then calculated the square root of the diagonal element of $V(b_1)$.

3. Result of Removal of Seasonal (cyclic) Influence. At first, we set $X_1 = \{x_1, x_2, x_3, x_4\}$. Then, we calculated the Least Squares Estimation of the global temperature Y over X_1 in order to get the de-seasonalized values of global temperature $Y^* = Y - \hat{Y}$ (Fig. 4), after calculating $Q = X_1' X_1$, $b_1^* = Q^{-1} X_1' Y$ (Table 9), and $\hat{Y} = X_1 b_1^*$.

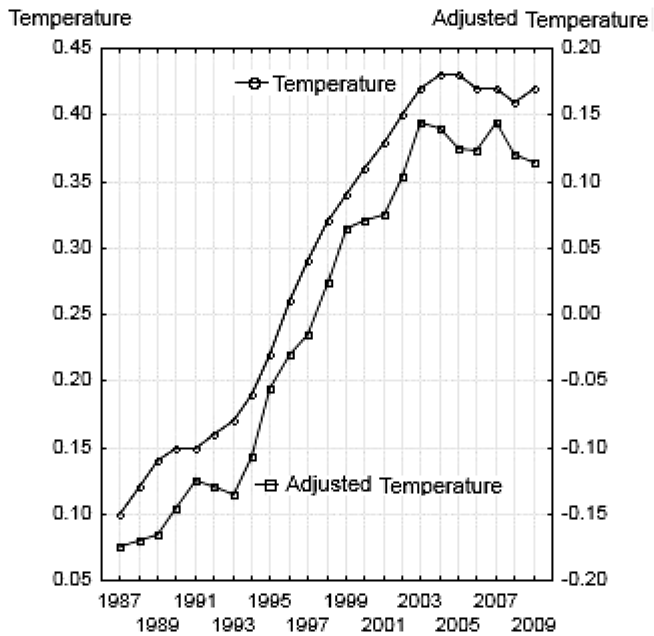

Fig. 4. Global temperatures with and without seasonal adjustment (Y^* and Y)

Table 9. Comprting b_1^* : $b_1^* = Q^{-1} X_1' Y$

Variable	x_1	x_2	x_3	x_4
Coefficient	0,27500	0,29000	0,30500	0,29600

And then, we implemented the Least Squares Estimation of X_4 on the dummy variables X_1, to get the coefficient $F = (X_1' X_1)^{-1} X_1 X_4$ (Table 10) and residuals $X_4^* = X_4 - X_1 F$ (Fig. 5), to de-seasonalize X_4 (to remove the influence of the cyclic change of the distance between Moon and Earth from X_4). ($X_4 = \{z_2, z_3\}$, where z_2 is the measured global CO_2, and z_3 is $1/r^2$). Table 11 shows descriptive statistics of de-seasonalized values of global temperature, CO_2 and $1/r^2$.

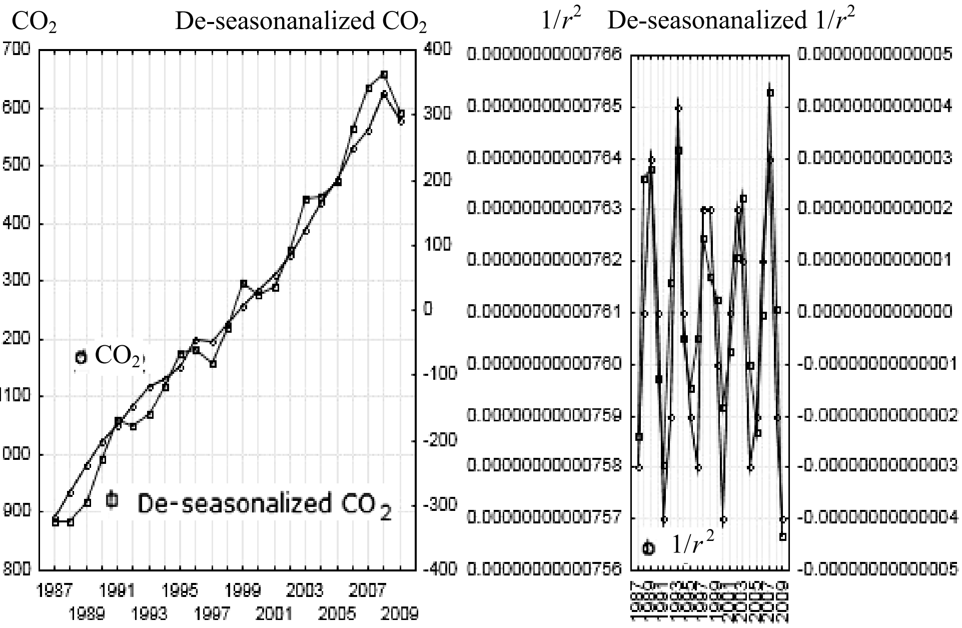

Fig. 5. Values of CO_2 and $1/r^2$ after removal of the influence of the cyclic change (X_4 and X_4^*)

T a b l e 10. Coefficient F

Variable	CO_2	$1/r^2$
x_1	$1{,}21717 \cdot 10^3$	$7{,}60012 \cdot 10^{-12}$
x_2	$1{,}26083 \cdot 10^3$	$7{,}58701 \cdot 10^{-12}$
x_3	$1{,}27717 \cdot 10^3$	$7{,}61690 \cdot 10^{-12}$
x_4	$1{,}25140 \cdot 10^3$	$7{,}61857 \cdot 10^{-12}$

T a b l e 11. Descriptive statistics of de-seasonalized values of global temperature, CO_2 and $1/r^2$

Variable	De-seasonalized global temperature, °C	De-seasonalized CO_2, mil. tons	De-seasonalized $1/r^2$, km
Mean	$-3{,}64431 \cdot 10^{-10}$	$-5{,}80497 \cdot 10^{-5}$	$1{,}13682 \cdot 10^{-20}$
Standard deviation	0,12072	$2{,}13017 \cdot 10^2$	$2{,}13368 \cdot 10^{-14}$
Minimum	-0,17500	$-3{,}25833 \cdot 10^2$	$-4{,}33389 \cdot 10^{-14}$
Maximum	0,14500	$3{,}65167 \cdot 10^2$	$4{,}30634 \cdot 10^{-14}$
Skewness	-0,19145	0,15571	$9{,}76442 \cdot 10^{-2}$
Kurtosis	1,28736	1,79673	2,29807
Valid number of observations	23	23	23

Then we implemented the Least Squares Estimation of the de-seasonalized global temperature Y^* on the de-seasonalized explanatory variables X_4^*, from which the influence of cyclic movement of Moon has been removed. Table 12 shows the result of the Least Squares Estimation.

Table 12. Result of the Least Squares Estimation of Y^* on the de-seasonalized explanatory variables X_4^*

Parameter		Coefficient	Standard error
Intercept *	1st cycle	-0,77339	$5,16897 \cdot 10^{-2}$
	2nd cycle	-0,78101	$5,16897 \cdot 10^{-2}$
	3rd cycle	-0,77630	$5,16897 \cdot 10^{-2}$
	4th cycle	-0,77163	$5,66233 \cdot 10^{-2}$
CO_2		$5,33726 \cdot 10^{-4}$	$4,29402 \cdot 10^{-5}$
$1/r^2$		$5,24677 \cdot 10^{10}$	$4,28696 \cdot 10^{11}$

*To get the coefficients of intercepts for 4 periods of the cycle, at first we calculated $b_1{}^* = Q^{-1}X_1Y$, and then, $b_1 = b_1{}^* - Fb_2$, where b_2 is the coefficients of CO_2 and $1/r^2$ in Table 12. And, to get the standard errors for the intercepts, we calculated $N_1 = X_1Q^{-1}Y$ and $e_1 = (I_1 - N_1)Y$, $V(b_1) = \frac{e_1'e_1}{n-k}Q^{-1}$, and then calculated the square root of the diagonal element of $V(b_1)$.

ANALYSIS OF THE RESULTS

In this research, we investigated influence of the trend (time) and the cyclic change of the distance between Moon and Earth. For this purpose, we set dummy binary variables, which replaced the intercept vectors of the Classical Regression Model, and then we calculated the coefficients of the Least Square Estimations between these binary variables and the global temperature and the explanatory variables (CO_2 and $\frac{1}{r^2}$), and then, we calculated expected influences to those variables from each of the trend (time) and the cyclic change; and then, we subtracted those expected values from the original values of the variables, in order to make the de-trended variables and the de-seasonalized variables. As the result, we observed that the coefficient of $1/r^2$ is larger than the coefficient of CO_2. This observation suggests that there is the influence of Moon's gravitational-wave to Earth's global temperature, which we also observed in our previous research [1].

In addition, the Maximum Likelihood Estimation shows almost as same values of the coefficients as in the Least Squares Estimation of Linear Classical Regression Model [1], while their standard errors of the coefficients are larger than those of the Least Squares Estimation. These differences of the standard errors are due to the difference of the algorithm of these two approaches: the values of the standard errors of the Least Squares Estimation are algebraically calculated, while the values of the Maximum Likelihood Estimation were searched numerically.

With the trend removal, the coefficient of global CO_2 became negative, because this process deformed the values of the global temperature and CO_2, as Fig. 2, 3 show.

Table 13, Table 14 and Table 15 show the results of the analysis, including the results of our previous research [1]. Among these 7 models in Table 13, 14, 15, the Pure Heteroskedasticity model and the Cobb-Douglas model (non-linear) show the larger coefficient of CO_2 than to the coefficient of $\frac{1}{r^2}$. Here, the Pure Heteroskedastic model assumes uneven distribution of the data, although the deviations of the values do not reflect the uneven distributions of global temperature and CO_2, which are as shown from Fig. 2–5; therefore, this model does not describe the data correctly. Also, the Cobb-Douglas model does not describe the distributions of the global temperature and CO_2, which are almost linearly distributed as Fig. 2, 3 show. On the other hand, the values of $\frac{1}{r^2}$ are on a same curve, therefore they are neither uniformly distributed, nor unevenly distributed; and, we conclude that this characteristic of $\frac{1}{r^2}$ gives the relatively large standard errors of the coefficient of $\frac{1}{r^2}$.

Table 13. Comparison of calculated coefficients and standard errors

Variable and coeficients		Classical Regression [1]	Maximum Likelihood Estimation	Trend removal	Removal of seasonal (cyclic) influence
Coefficient	Intercept	-1,17863	-1,17897	See Table 6	See Table 12
	CO_2	$5{,}33150 \cdot 10^{-4}$	$5{,}32537 \cdot 10^{-4}$	$-1{,}15220 \cdot 10^{-5}$	$5{,}33726 \cdot 10^{-4}$
	$1/r^2$ *	$1{,}05537 \cdot 10^{11}$	$1{,}05675 \cdot 10^{11}$	$1{,}89763 \cdot 10^{9}$	$5{,}24677 \cdot 10^{10}$
Standard error	Intercept	2,77830	$1{,}67861 \cdot 10^{3}$	See Table 8	See Table 12
	CO_2	$4{,}27704 \cdot 10^{-5}$	$2{,}95105 \cdot 10^{-2}$	$3{,}19869 \cdot 10^{-5}$	$4{,}29402 \cdot 10^{-5}$
	$1/r^2$ **	$3{,}64933 \cdot 10^{11}$	$2{,}19071 \cdot 10^{14}$	$5{,}33765 \cdot 10^{9}$	$4{,}28696 \cdot 10^{11}$
* : **		1 ; 3,46	1 ;: 2070	1;: 2,81	1 ;: 8,17

Table 14. Coefficients and standard errors of the coefficients in Generalized Classical Regression Model [1]

Variable	Pure Heteroskedasticity		First-Order Autoregressive Process	
	Coefficient	Standard error	Coefficient	Standard error
for 1 (x_1)	-9,72055	22,91283	0,37507	0,78957
for Carbon dioxide (x_2)	0,94202	$7{,}55710 \cdot 10^{-2}$	$1{,}36503 \cdot 10^{-5}$	$1{,}17412 \cdot 10^{-4}$
for $\frac{1}{r^2}$ (x_3)	$2{,}18557 \cdot 10^{-2}$ *	$7{,}55709 \cdot 10^{-3}$ **	$6{,}61708 \cdot 10^{9}$ *	$9{,}71690 \cdot 10^{10}$ **
* : **	2,89 : 1		1 : 14,7	

Table 15. Coefficients of Cobb-Douglas model, $y = b_1 x_2^{b2} x_3^{b3}$ [1]

Coefficients	Estimated coefficient	Standard error
b_1, coefficient of 1	0,000103	0,02761
b_2, coefficient of x_2	2,126546	0,23431
b_3, coefficient of x_3	0,283107 *	10,62035 **

* :** 1 : 37,5.

Note: y : global temperature; x_2 : carbon-dioxide; $x_3 : \frac{1}{r^2}$.

CONCLUSION AND RECOMMENDATION

We have examined the potential influence of Moon's gravitational-wave to Earth's global temperature, in comparison with global CO_2, using 7 mathematical models for the empirical analysis. As the result, the influence of Moon's gravitational-wave was found to have some relation with Earth's temperature rise, with the Least Squares Estimation of Classical Regression Model, the First-Order Autoregressive Process of Generalized Classical Regression Model, the Maximum Likelihood Estimation, the Least Squares Estimation after the removal of trend (time), and after the removal of seasonal (cyclic) influence; while, the assumption of Pure Heteroskecasticity and Cobb-Douglas model (non-linear) are not appropriate for this analysis, in regard to the linearly distributed Earth's global temperature and global CO_2 in time series.

The further study is needed to identify the meaning of the uncertain relation between the inverse of squared distance between Moon and Earth and Earth's temperature rise.

REFERENCE

1. *Matsuki Y.* Empirical analysis of moon's gravitational wave and earth's global warming / Y. Matsuki, P.I. Bidyuk // System Research & Information Technology. — 2018. — N 1. — P. 107–118.
2. *UK Department* of Energy and Climate Change (DECC). — Available at: http://en.openei.org/datasets/dataset/b52057cc-5d38-4630-8395-b5948509f764/resource/f42998a9-071e-4f96-be52-7d2a3e5ecef3/download/england.surface.temp1772. 2009.xls
3. *Boden T.A.* Global Regional and National Fossil-Fuel CO2 Emissions / T.A. Boden, G. Marland, R.J. Andres. — Available at: cdiac.orbl.gov/trends/emits/tre_glob.html cdiac.ornl.gov/trends/emits/tre_glob.html (last access, 8 August 2017)
4. *Moon* Distance Calculator – How Close is Moon to Earth? — Available at: https://www.timeanddate.com/astronomy/moon/distance.html?year=1987&n=367.

Received 02.05.2018

From the Editorial Board: the article corresponds completely to submitted manuscript.