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FUZZY PORTFOLIO OPTIMIZATION PROBLEM UNDER UNCERTAINTY CONDITIONS WITH APPLICATION OF COMPUTATIONAL INTELLIGENCE METHODS

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Abstract. The problem of constructing an optimal securities portfolio under uncertainty is considered along with the direct and dual problems of fuzzy portfolio optimization. The modified fuzzy portfolio optimization problem is also suggested under a constraint on portfolio volatility. In the dual problem, the portfolio structure is determined, which provides the minimum risk level at the specified profitability level. The use of forecasting share prices for the portfolio model was suggested to support the validity of decisions on the portfolio structure and to reduce the risks. The share price data for the portfolio optimization system are forecasted using Fuzzy Group Method of Data Handling (FGMDH). The experimental studies of the suggested fuzzy models were carried out, and a comparison with the Markowitz model was performed on a stock market. As the result of this work, the foundations of the theory of fuzzy portfolio optimization are built on the basis of the theory of fuzzy sets and an effective forecasting method.

Keywords: fuzzy portfolio, modified fuzzy portfolio optimization model, forecasting, share prices, fuzzy GMDH.

INTRODUCTION

The problem of investment in securities had arisen with appearance of the first stock markets. Careful processing and accounting of investment risks have become an integral and important part of the success of each company. However, more and more companies have to make decisions under uncertainty, which may lead to undesirable results. Particularly serious consequences may have the wrong decisions at long-term investments. Therefore, early detection, adequate and the most accurate assessment of risk is one of the crucial problems of modern investment analysis.

The beginning of modern investment theory was put in the article H. Markowitz, “Portfolio Selection”, which was published in 1952. In this article mathematical model of optimal portfolio of securities was first proposed. Methods of constructing such portfolios under certain conditions are based on theoretical and probabilistic formalization of the concept of profitability and risk. For many years the classical theory of Markowitz was the main theoretical tool for optimal investment portfolio construction, after which most of the novel theories were only modifications of the basic theory.

New approach in the problem of investment portfolio construction under uncertainty is connected with fuzzy sets theory, created about half a century ago in the fundamental work of Lotfi Zadeh [6]. By using fuzzy numbers in the forecast share prices a decision-making person was not required to form probability estimates.

The application of fuzzy sets technique enables to create a novel theory of fuzzy portfolio optimization under uncertainty and risk deprived of drawbacks of classical portfolio theory.

The main source of uncertainty is changing stock prices of securities at the stock market as the decision on portfolio is based on current stock prices while the implementation of portfolio is performed in future and portfolio profitability depends on future prices which are unknown at the moment of decision making. Therefore that to raise the reliability of decision concerning portfolio and cut possible risk it' s needed to forecast future prices of stocks. For this the application of inductive modeling method, so-called Fuzzy Group Method of Data Handling (FGMDH) seems to be very perspective.

The goals of this paper are to present and analyze the results in fuzzy portfolio optimization theory, to consider and analyze so-called classic and new direct problems of fuzzy portfolio optimization, to estimate the application of FGMDH for stock prices forecasting and to carry out experimental investigations for estimation of the efficiency of the elaborated theory.

DIRECT PROBLEM OF FUZZY PORTFOLIO OPTIMIZATION

Problem statement

Let us consider a share portfolio of N components and its expected behaviour at time interval $[0,T]$. Each of a portfolio component $i=\overline{1,N}$ at the moment T is characterized by its financial profitability r_i (evaluated at a point T as a relative increase in the price of the asset for this period) Assume the capital of the investor be equal 1. The problem of share portfolio optimization consists in a finding of a vector of share prices distribution in a portfolio $x=\{x_i\}$ $i=\overline{1,N}$ maximizing the expected income at the set risk level.

In process of practical application of Markowitz model its drawbacks were detected [2, 3]:

- The hypothesis about normality of profitability distributions usually isn't proved in practice.
- Stationarity of price processes is not always valid in practice.
- At last, the risk of stocks is considered as a covariance or its volatility, therefore the decrease in profitability in relation to the expected value, and profitability increase are estimated in this model absolutely equally, while for the owner of securities these events are absolutely different.

These weaknesses of Markowitz theory caused necessity of development essentially new approach for definition of an optimum investment portfolio.

Let's consider the main principles and ideas of a fuzzy method for portfolio optimization [1, 2, 3, 4].

The risk of a portfolio is not its volatility (covariance), but possibility that expected profitableness of a portfolio will appear below some pre-established planned value r^*.

Correlation of stock prices in a portfolio is not considered and not taken into account.

Profitability of each security is not random, but fuzzy number. Similarly, restriction on extremely low level of profitability can be both usual scalar and fuzzy number of any kind. Therefore, optimize a portfolio in such statement may mean, in that specific case, the requirement to maximize expected profitability of a portfolio in time moment T at the fixed risk level.

Profitability of a security on termination of ownership term is expected to be equal r and lies in a certain range.

For i -th security let's denote:

$\bar{r}_i$ — the expected profitability of the i-th security;

r_{1i} — the lower border of profitability of the i-th security;

r_{2i} — the upper border of of the i-th security;

$r_i = (r_{1i}, \bar{r}_i, r_{2i})$ — profitability of the i-th security is a triangular or Gaussian fuzzy number.

Then profitability of a portfolio is equal:

$$r = \left(r_{\min} = \sum_{i=1}^{N} x_i r_{1i};\ \bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i;\ r_{\max} = \sum_{i=1}^{N} x_i r_{2i} \right), \qquad (1)$$

where x_i is the weight of i -th security in a portfolio (its portion), for which the following conditions holds

$$\sum_{i=1}^{N} x_i = 1, \quad 0 \le x_i \le 1. \qquad (2)$$

Critical level of profitability of a portfolio at the moment of T given by an investor may be non-fuzzy or fuzzy number $r^* = (r_1^*, \bar{r}^*, r_2^*)$ as well.

Fuzzy-set portfolio optimization with triangular membership functions

To determine the structure of a portfolio which provides the maximum profitability at the set risk level, it is required to solve the following problem [2, 3].

Find $\{X_{\text{opt}}\} = \{X\}$, for which $r \to \max$, under condition $\beta(X) = \text{const}$, where r is a portfolio profitability, β is a desired risk, vector X satisfies (2).

In [2, 3] the following expression for risk β (x) was obtained for critical value r^*

$$\frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \bar{r}_i - r^* \right) \ln \left(\frac{\sum_{i=1}^{N} x_i \bar{r}_i - r^*}{\sum_{i=1}^{N} x_i \bar{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) = \beta. \qquad (3)$$

Taking into account also that profitability of a portfolio is equal to:

$$r = \left(r_{\min} = \sum_{i=1}^{N} x_i r_{1i};\ \bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i;\ r_{\max} = \sum_{i=1}^{N} x_i r_{2i} \right),$$

where $(r_{1i}, \bar{r}_i, r_{2i})$ is the profitability of *i*-th security, we obtain the following optimization problem [2, 3]:

$$\bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i \to \max; \qquad (4)$$

$$\beta = \text{const}; \qquad (5)$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N}. \qquad (6)$$

At a risk level variation β 3 cases are possible: $\beta = 0$, $\beta = 1$ and $0 < \beta < 1$. Consider in detail the last case $0 < \beta < 1$.

This case is possible when $\sum_{i=1}^{N} x_i r_{1i} \leq r^* \leq \sum_{i=1}^{N} x_i \bar{r}_i$ or when $\sum_{i=1}^{N} x_i \bar{r}_i \leq r^* \leq$ $\leq \sum_{i=1}^{N} x_i r_{i2}$.

Assume $\sum_{i=1}^{N} x_i r_{i1} \leq r^* \leq \sum_{i=1}^{N} x_i \bar{r}_i$ .Then using (3) the problem (4)–(6) is reduced to the following nonlinear programming problem [2–5]:

$$\bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i \to \max. \qquad (7)$$

Under conditions

$$\frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \bar{r}_i - r^* \right) \ln \left(\frac{\sum_{i=1}^{N} x_i \bar{r}_i - r^*}{\sum_{i=1}^{N} x_i \bar{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) = \beta; \qquad (8)$$

$$\sum_{i=1}^{N} x_i r_{i1} \leq r^*; \qquad (9)$$

$$\sum_{i=1}^{N} x_i \bar{r}_i > r^*; \qquad (10)$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N}. \qquad (11)$$

Let $\sum_{i=1}^{N} x_i \bar{r}_i \leq r^* \leq \sum_{i=1}^{N} x_i r_{i2}$. Then the problem (9)–(11) is reduced to the following nonlinear programming problem [2–5]:

$$\bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i \to \max; \qquad (12)$$

$$\left(r^* - \sum_{i=1}^{N} x_i r_{i1}\right) -$$

$$-\left(r^* - \sum_{i=1}^{N} x_i \bar{r}_i\right) * \ln\left(\frac{r^* - \sum_{i=1}^{N} x_i \bar{r}_i}{\sum_{i=1}^{N} x_i \bar{r}_{i2} - \sum_{i=1}^{N} x_i r_{i1}}\right) * \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta \; ; \quad (13)$$

$$\sum_{i=1}^{N} x_i r_{i2} > r^* ; \quad (14)$$

$$\sum_{i=1}^{N} x_i \bar{r}_i \le r^* ; \quad (15)$$

$$\sum_{i=1}^{N} x_i = 1, \; x_i \ge 0, \; i = \overline{1, \tilde{N}} . \quad (16)$$

The penalty functions algorithm of minimization is suggested to find the solution of problems (7)–(11) and (12)–(16). Assume both problems (7)–(11) and (12)–(16) be solvable. Then structure of a required optimumal portfolio will be such vector $X = \{x_i\}$ $i = \overline{1, N}$ being the solution of problems (7)–(11) or (12)–(16) for which the criterion value will be greater.

Modified fuzzy portfolio optimization under constraint on volatility

Analysis of fuzzy portfolio optimization model (7)–(10) and previous experimental results [2–5] has shown that the constraint on risk in fuzzy portfolio model doesn't make real restriction of profitability and acts in the same way as criterion: the less risk the greater is profitability.

It leads to some bias in portfolio content, usually the portion of the most profitable share takes the greatest value and practically doesn't change with risk variation. Therefore it was suggested to modify fuzzy portfolio model by introducing the constraint on portfolio volatility Δ_{giv} :

$$\sum_{i=1}^{N} x_i (r_{i2} - r_{i1}) \le \Delta_{\text{giv}};$$

where

$$\Delta_{\text{giv}} = \text{var}, \;\; \Delta_{\text{giv}} : (r_{i2} - r_{i1}) \min \div (r_{i2} - r_{i1}) \max .$$

Using this transformation we obtain two linear programing problems presented below:

a) for the case $\sum_{i=1}^{N} x_i r_{i1} \le r^* < \sum_{i=1}^{N} x_i \tilde{r}_i,$

LP-problem takes the following form:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max;$$

$$\sum_{i=1}^{N} x_i (r_{i2} - r_{i1}) \le \Delta_{\text{giv}};$$

$$\sum_{i=1}^{N} x_i r_{i1} \le r^*;$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i \ge r^*;$$

$$\sum_{i=1}^{N} x_i = 1, \;\; x_i \ge 0, \;\; i = \overline{1, N};$$

$$\Delta_{\text{giv}} \in [(r_2 - r_1)\min \div (r_2 - r_1)\max];$$

b) for the case $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* < \sum_{i=1}^{N} x_i r_{i2}$, we obtain the following LP-problem:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max;$$

$$\sum_{i=1}^{N} x_i (r_{i2} - r_{i1}) \le \Delta_{\text{giv}};$$

$$\sum_{i=1}^{N} x_i r_{i2} \ge r^*;$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i \le r^*;$$

$$\sum_{i=1}^{N} x_i = 1, \;\; x_i \ge 0, \;\; i = \overline{1, N},$$

where

$$\Delta_{\text{giv}} \in [\,(r_2 - r_1)\min \div (r_2 - r_i)\max].$$

If both these LP-problems are solvable then the structure optimal invest portfolio $x = \{x_i\}$ $i = \overline{1, N}$ will be determined for which the portfolio profitability will be greater.

THE ANALYSIS AND COMPARISON OF THE RESULTS BY MARKOWITZ AND FUZZY-SETS MODELS

For comparative analysis of investigated methods the following corporations Apple INC, Cisco, Intel, Microsoft, Visa. were chosen whose shares are presented in stock- exchange NYSE and used for Dow Jones index calculation. The information about share prices of these corporations were taken from site https://www.nyse.com. The data about share prices were taken in the period from 01.01.2018 to 29.06.2018 year, in a whole the period of 125 days.

The obtained data from NYSE contained the information about opening and closing share prices of the chosen companies.

For fuzzy portfolio optimization the calculated profitability range (region) and expected profitability value were previously calculated for each share. In Markowitz model expected profitability of a share is calculated as a mean $m = M\{r\}$ and risk of an asset is considered as a dispersion of the profitability value.

Using this data for all stocks the following indicators were calculated: 1) mean profitability, 2) left bound of profitability $r_{\min}$, 3) right bound $r_{\max}$ 4) profitabilty range (volatility) Δзад and variance σ^2. These data are presented in the table 1.

Table 1. Shares Profitability, volatility, and variance

N	Company	$\tilde{r}$, %	$r_{\min}$	$r_{\max}$	Δ_{giv}	σ^2
1	APPLE INC	0,0735	-0,9585	0,8700	1,8285	1,6015
2	CISCO SYS INC	0,0932	-0,8681	0,9564	1,8245	1,7895
3	INTEL CORP	0,0643	-1,3545	1,1903	2,5348	2,8340
4	MICROSOFT CORP	0,1183	-0,9063	0,8227	1,7290	1,3972
5	VISA INC	0,1255	-0,8200	0,6976	1,5176	1,1478

Portfolio optimization for Markowitz model

Construct mathematical model of Markowitz using (7)–(11). Number of shares in a portfolio equals to 5. Choose the initial risk level $\beta = 0,5$. Then at each step increasing risk by 0,5 consider and solve models for risk value from 0,5 to 4,5. For its solution penalty functions method was applied and software kit in program language C# was developed. The obtained results for risk values $\beta \in (0,5 \div 4,5)$ with step 0,5 are presented in the table 2.

Table 2. Portfolio constructed by Markowitz model for different risk levels

N	Risk β	Profitability	Shares				
			APPLE INC	CISCO SYS INC	INTEL CORP	MICRO SOFT CORP	VISA INC
1	0,5	0,0928	0,2491	0,1491	0,0745	0,0745	0,3745
2	1,0	0,0940	0,2529	0,1529	0,0767	0,0764	0,3764
3	1,5	0,0969	0,2581	0,1513	0,0833	0,0830	0,3830
4	2,0	0,0981	0,2643	0,1643	0,0908	0,0908	0,3908
5	2,5	0,0988	0,2635	0,1635	0,0862	0,0862	0,3862
6	3,0	0,1005	0,4296	0,0294	0,1125	0,0875	0,3875
7	3,5	0,1022	0,3852	0,0852	0,1137	0,0862	0,3862
8	4,0	0,1022	0,3852	0,0852	0,1137	0,0862	0,3862
9	4,5	0,1022	0,3852	0,0852	0,1137	0,0862	0,3862

The corresponding dependence "profitablility/risk" for Markowitz model based on this data is presented in the fig. 1.

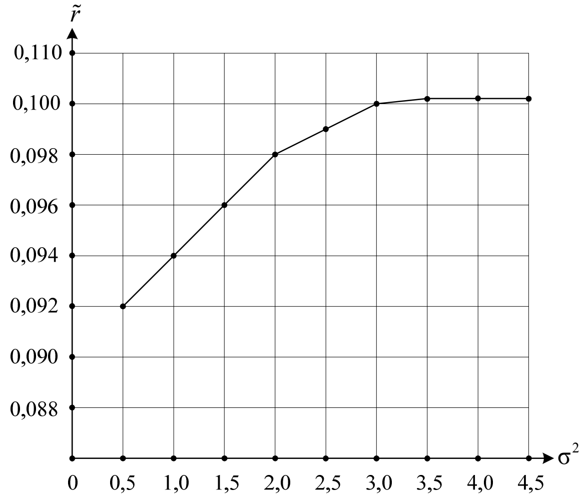

Fig. 1. Dependence profitability versus risk for optimal portfolio by Markowitz

Fuzzy portfolio optimization model

As it was stated previously for fuzzy portfolio with risk variation there are three different cases. Consider the case 0 < β < 1 when $\sum_{i=1}^{N} x_i r_{i1} \leq r^* < \sum_{i=1}^{N} x_i \tilde{r}_i$.

Let solve fuzzy portfolio problem for this case. The corresponding expression for risk $\beta(X)$ is presented in (8). As critical value for risk take $r^* = 0,08$.

The following model is non-linear programing problem. For its solution penalty functions method was applied. This problem was solved using developed software kit in language C# for risk values $\beta:(0,1 \div 0,9)$ with step 0,1 and the results are presented in the table 3.

Table 3. Optimal portfolios for fuzzy model for different risk values

N	Risk β	Profitability	Assets				
			APPLE INC	CISCO SYS INC	INTEL CORP	MICROSOFT CORP	VISA INC
1	0,1	0,0983	0,2406	0,1343	0,0750	0,1375	0,3750
2	0,2	0,0973	0,2355	0,1355	0,0722	0,1361	0,3722
3	0,3	0,0959	0,2307	0,1307	0,0696	0,1348	0,3696
4	0,4	0,0945	0,2261	0,1261	0,0672	0,1336	0,3672
5	0,5	0,0932	0,2271	0,1217	0,0648	0,1324	0,3648
6	0,6	0,0904	0,2154	0,1154	0,0577	0,1288	0,3577
7	0,7	0,0893	0,2116	0,1116	0,0558	0,1279	0,3558
8	0,8	0,0882	0,2079	0,1079	0,0539	0,1269	0,3539
9	0,9	0,0872	0,2045	0,1045	0,0522	0,1261	0,3522

The corresponding dependence profitability/risk for fuzzy model is presented in the fig. 2.

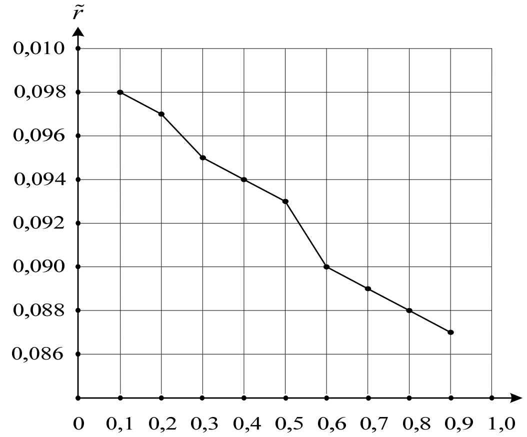

Fig. 2. Dependence "optimal profitability versus risk for fuzzy portfolio"

From this figure it follows that with risk value increase the profitability of optimal fuzzy portfolio drops. Compare these results with Markowitz model (fig. 1).

Dependences of expected profitability on degree of the portfolio risk, received by the above specified methods, are practically opposite. The reason of such result is the different understanding of a portfolio risk. In the fuzzy-set method the risk is understood as a situation when expected profitability of a portfolio falls below the critical level r^*set by an investor, so with decrease of expected profitability risk of portfolio investments to be less than the critical value, increases .

In Markowitz model the risk is considered as the degree of expected profitability variability of a portfolio, in both cases of smaller and greater profitability that contradicts the common sense. The various understanding of portfolio risk level is also the reason of difference of a portfolio structure, received by different methods.

The structures of an optimum portfolio which are obtained by both methods for the same risk levels are also quite different.

New fuzzy portfolio optimization model with constraint on volatility

In this problem portfolio volatility was calculated as difference between maximal and minimal profitability for all the shares. In order to implement this approach the data of all shares prices in portfolio in the period since 01.02.2018 to 02.08. 2018 were taken and for all shares the mean profitability, low and upper bounds and volatility were calculated. The obtained results are presented in the table 4.

Table 4. Shares profitability and volatility

N	Company	$\tilde{r}$, %	r_{min}	r_{max}	Δ
1	APPLE INC	0,0373	-1,0950	0,9000	1,9550
2	CISCO SYS INC	0,3982	-1,1214	1,0653	2,1867
3	INTEL CORP	0,1865	-1,5118	1,4362	2,9481
4	MICROSOFT CORP	0,2428	-0,8635	0,7402	1,6037
5	VISA INC	0,1969	-1,0242	0,7914	1,8156

From analysis of the results in the table 4 ift follows that the greatest mean profitability have shares of CISCO SYS INC, and the least volatility is for shares of MICROSOFT CORP.

After solution of the corresponding LP-problems optimal portfolio profitability in dependence of volatility was determined which is presented in the table 5.

Table 5. Dependence of mean portfolio profitability versus volatility

N	Δ_{giv}	Portfolio volatility	Shares				
			APPLE INC	CISCO SYS INC	INTEL CORP	MICROSOFT CORP	VISA INC
1	1,7	0,269	0,000	0,165	0,000	0,835	0,000
2	1,8	0,295	0,000	0,337	0,000	0,663	0,000
3	1,9	0,322	0,000	0,508	0,000	0,492	0,000
4	2,0	0,349	0,000	0.680	0,000	0.320	0,000
5	2,1	0,376	0,000	0,851	0,000	0,149	0,000
6	2,2	0,3982	0,000	1,000	0,000	0,000	0,000

Using these results the chart of portfolio profitability for optimal portfolio is shown in the fig. 3.

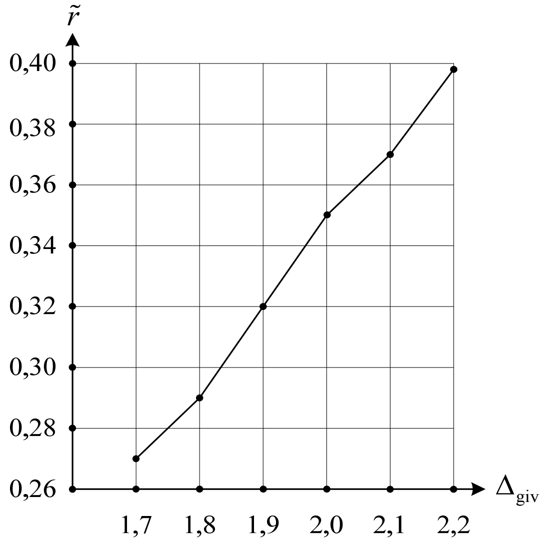

Fig. 3. Dependence of profitability versus volatility for optimal portfolio

From this chart it follows that with increase of volatility the mean profitability nof fuzzy portfolio also grows. As we see this dependence well matches to Markowitz portfolio model.

THE DUAL PORTFOLIO OPTIMIZATION PROBLEM

Fuzzy portfolio optimization

Now consider the portfolio optimization problem dual to the problem (7)–(11) [7].

To minimize $\beta(x)$; under conditions

$$\bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i \geq r^* ;$$

$$\sum_{i=1}^{N} x_i = 1 , \ x_i \geq 0 ,$$

where expression for $\beta(x)$ is given by (3)

Optimality Conditions for Dual Fuzzy Portfolio Problem

As it was proved in [8] function $\beta(x)$ is convex therefore the dual portfolio problem (14)–(16) is a convex programming problem. Taking into account that constraints (16) are linear compose Lagrangian function:

$$L(x, \lambda, \mu) = \beta(x) + \lambda(r^* - \sum_{i=1}^{N} x_i \bar{r}_i) + \mu\left(\sum_{i=1}^{N} x_i - 1\right).$$

The optimality conditions by Kuhn–Tucker are such:

$$\frac{\partial L}{\partial x_i} = \frac{\partial \beta(x)}{\partial x_i} - \lambda \bar{r}_l + \mu \geq 0, \ 1 \leq i \leq N ;$$

$$\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{N} x_i \bar{r}_l + r^* \leq 0, \ \frac{\partial L}{\partial \mu} = \sum_{i=1}^{N} x_i - 1 = 0 ,$$

and conditions of complementary slackness

$$\frac{\partial L}{\partial x_i} x_i = 0, \ 1 \leq i \leq N,. \ \frac{\partial L}{\partial \lambda}\lambda = \lambda\left(-\sum_{i=1}^{N} x_i \bar{r}_i + r^*\right) = 0, \ x_i \geq 0, \ \lambda \geq 0 ,$$

where $\lambda \geq 0$ and μ are indefinite Lagrange multipliers.

This problem may be solved by standard methods of convex programming, for example method of feasible directions or method of penalty functions.

THE APPLICATION OF FGMDH FOR STOCK PRICES FORECASTING IN PORTFOLIO OPTIMIZATION PROBLEM

Consider the results of experimental investigations of the developed models and methods of fuzzy portfolio optimization. The profitability of leading companies at NYSE in the period from 03.09.2013 to 17.01.2014 were used as the input data in experimental investigations. The companies included: Canon Inc. (CAJ), McDonald's Corporation (MCD), PepsiCo, Inc. (PEP), The Procter & Gamble Company (PG), SAP AG (SAP).

For forecasting it was suggested to use fuzzy GMDH method with triangular membership functions, linear partial descriptions, training sample of 70%, forecasting for 1 step.

The fuzzy GMDH allows to construct forecasting model using experimental data automatically without participation of an expert. Besides, it may work under uncertainty conditions with fuzzy input data or data given as intervals [9].

The next profitability values real and forecasted by fuzzy GMDH on 17.01.2014 were obtained (table 6):

Table 6. The profitability of shares on date 17.01.2014 (real and forecasted), %

Companies	Profitability				MSE test sample	MAPE test sample
	Real value	Low bound	Forecasted value	Upper bound		
CAJ	-1,270	-1,484	-1,246	-1,008	2,2068	0,0295
MCD	-0,105	-0,347	-0,118	0,111	2,5943	0,0091
PEP	0,206	0,001	0,242	0,483	3,0179	0,0177
PG	0,162	0,041	0,170	0,299	1,6251	0,0197
SAP	0,843	0,675	0,867	1,059	2,3065	0,0164

Thus, as the result of application of FGMDH the shares profitability values were forecasted to the end of 20-th week (17.01.2014).

Let the critical profitability level be 0,7%. Varying the risk level we obtain the following results at the end of 20-th week (17.01.2014) for triangular MF. The results are presented in the table 7 and the dependence "optimal profitability versus risk" is shown in the fig. 4.

Table 7. Parameters of the optimal portfolio with critical level $r^* = 0{,}7\%$

Low bound	Expected profitability	Upper bound	Risk level
0,55133	0,74591	0,94049	0,2
0,53462	0,72954	0,92446	0,25
0,51544	0,71084	0,90624	0,3
0,51894	0,71431	0,90968	0,35
0,5045	0,70018	0,89587	0,4
0,50877	0,70425	0,89973	0,45
0,522	0,71731	0,91262	0,5
0,50197	0,69752	0,89308	0,55
0,46358	0,66014	0,8567	0,6

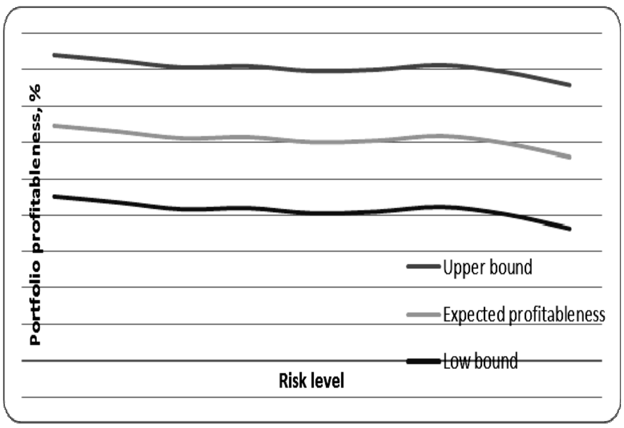

Fig. 4. Dependence of the expected portfolio profitability versus risk level for triangular MF

As one can see in the fig. 4, the dependence profitability - risk has descending type, the greater is the risk the lesser is profitability which is opposite to classical probabilistic method. When the expected profitability decreases, the risk grows.

The profitability of the real portfolio is 0,7056%.This value falls in calculated corridor of profitability for optimal portfolio [0,58944, 0,78264, 0,97584] built with application of forecasting, indicating the high accuracy of the forecast (see table 7).

Let's consider the results obtained by solving the dual problem using triangular MF. In this case, the investor sets the rate of return and the problem is to minimize the risk.

The optimal portfolio is presented in table 8 and the dependence "critical level of return- risk" is shown in the fig. 5.

Table 8. Parameters of the optimal portfolio (dual task)

Low bound	Expected profitability	Upper bound	Risk level	Critical rate of return
0,58944	0,78264	0,97584	0,00025	0,6
0,59846	0,79141	0,98437	0,01468	0,65
0,61478	0,80735	0,99991	0,04973	0,7
0,6229	0,81531	1,00772	0,13347	0,75
0,63606	0,82822	1,02037	0,26399	0,8
0,64945	0,84181	1,03417	0,49937	0,85
0,63712	0,82933	1,02153	0,72631	0,86
0,63382	0,82612	1,01843	0,8333	0,87
0,62559	0,81805	1,01052	0,91214	0,88

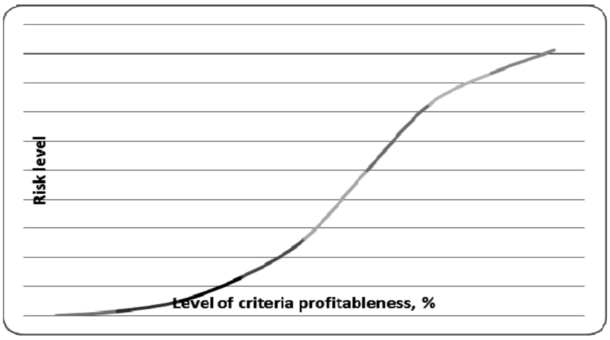

Fig. 5. Dependence of the risk level on a given critical return r^*

From these results one can see that the curve "dependence risk – given critical level of profitability" has the ascending character, because with the growth of the critical value of profitability r^* the probability that the expected profitability would be lower than a given critical value increases.

CONCLUSION

The problem of optimization of the investment portfolio under uncertainty is considered in this paper. The fuzzy-set approach for solving the direct and dual portfolio optimization problems was suggested and explored. The modified direct portfolio problem with constraint on volatility was suggested and investigated The results of solving the problems were presented. The optimal portfolios for the five assets at NYSE stock market were constructed and analyzed.

The problem of stock prices forecasting for portfolio optimization was also investigated. Inductive modelling method fuzzy GMDH was applied for stocks prices forecasting at NYSE stock market in the problem of fuzzy portfolio optimization. The application of fuzzy GMDH enables to decrease risk of the wrong decisions concerning portfolio content.

After analysis of experiments it was detected that the dependence "profitableness – risk" in the basic fuzzy model has descending type, the greater risk the lesser is profitableness that is opposite to classical probabilistic Markowitz model. In the modified fuzzy portfolio problem the dependence "profitable volatility" has ascending type like Markowitz model.

The dependence "risk versus given critical level of return" has ascending type, because with the growth of the critical level of profitability the probability that the expected profitability appears to be lower than a given critical value increases.

As the main result of this research the fundamentals of theory of fuzzy portfolio optimization under uncertainty have been developed free from drawbacks of the classic portfolio theory.

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ПРОБЛЕМА НЕЧІТКОЇ ПОРТФЕЛЬНОЇ ОПТИМІЗАЦІЇ В УМОВАХ НЕВИЗНАЧЕНОСТІ З ВИКОРИСТАННЯМ МЕТОДІВ ОБЧИСЛЮВАЛЬНОГО ІНТЕЛЕКТУ / О.Ю. Зайченко, Ю.П. Зайченко

Анотація. Розглянуто проблему побудови оптимального портфеля з цінних паперів в умовах невизначеності, а також пряму та двоїсту задачі портфельної оптимізації. Запропоновано нову постановку задачі нечіткої оптимізації портфеля за обмежень на волатильність. У двоїстій задачі визначається структура портфеля, яка забезпечує мінімум ризику за обмежень на заданий рівень дохідності. Для підвищення обгрунтованості рішень щодо структури портфеля та зменшення ризику запропоновано використання прогнозування цін акцій в моделі портфеля. Дані за цінами акцій прогнозуються з використанрням нечіткого МГУА. Проведено експериментальні дослідженрня запропонованих нечітких моделей та порівняння з моделлю Марковітца на ринку цінних паперів. У результаті роботи створено основи теорії нечіткої портфельної оптимізації на базі теорії нечітких множин та методу прогнозування.

Ключові слова: нечіткий портфель, модифікована модель нечіткої портфельної оптимізації, прогнозування цін акцій, нечіткий МГУА.

ПРОБЛЕМА НЕЧЕТКОЙ ПОРТФЕЛЬНОЙ ОПТИМИЗАЦИИ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ С ПРИМЕНЕНИЕМ МЕТОДОВ ВЫЧИСЛИТЕЛЬНОГО ИНТЕЛЛЕКТА / Е.Ю. Зайченко, Ю.П. Зайченко

Аннотация. Рассмотрены проблема построения оптимального портфеля из ценных бумаг в условиях неопределенности, а также прямая и двойственная задачи нечеткой портфельной оптимизации. Предложена новая постановка задачи нечеткой оптимизации портфеля с ограничением на волатильность портфеля. В двойственной задаче определяется структура портфеля, которая обеспечивает минимум риска при заданном уровне доходности. Для обеспечения обоснованности принимаемых решений по структуре портфеля и уменьшения риска предложено использовать прогнозирование цен акций в модели портфеля. Данные по ценам акций для системы портфельной оптимизации прогнозируются с использованием нечеткого МГУА. Проведены экспериментальные исследования предложенных нечетких моделей и сравнение с моделью Марковитца на рынке ценных бумаг. В результате работы построены основы теории нечеткой портфельной оптимизации на основе теории нечетких множеств и эффективного метода прогнозирования

Ключевые слова: нечеткий портфель, модифицированная модель нечеткой портфельной оптимизации, прогнозирование цен акций, нечеткий МГУА.