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MODELING OF CONTACT INTERACTION OF A HEATED PLANE RIGID ELLIPTICAL PUNCH WITH A TRANSVERSALLY ISOTROPIC ELASTIC HALF-SPACE

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Abstact. On the base of a rigorous mathematical model, the problem of the contact interaction of a heated flat punch of an elliptical section with a transversely isotropic elastic half-space is investigated. It is assumed that the half-space surface is the isotropy plane of a transversely isotropic material, and also that there is a smooth (without friction) contact. Expressions of contact stresses and displacements of a heated flat elliptical punch are found explicitly. In the form of a simple inequality, a condition for separating the elastic material from the surface of a flat elliptical punch is obtained. Numerical calculations are carried out. Contact interaction of a heated flat punch is studied taking into account the separation of material from the punch.

Keywords: mathematical model, contact interaction, elastic half-space, transversally-isotropic material, plane elliptical punch, heating, stress distribution, domain of material separation.

INTRODUCTION

Currently, methods for solving spatial problems of contact interaction for isotropic elastic bodies are quite well developed. Among the papers on this topic, classical monographs [1–5], as well as articles [6–8], can be noted. However, the solution of spatial contact problems for transversely isotropic bodies is associated with significant mathematical difficulties, since the initial system of equations for determining the stress state has a more complex structure. Contact problems of thermoelasticity for a transversely isotropic half-space were studied in [8–10] and others. An approach was used in [9, 10] that allows one to investigate problems only for a circular contact region. In [8], the contact problem of thermoelasticity is studied with a special distribution of the temperature field on the surface of the punch, which is proportional to the contact pressure under the paraboloidal punch. In the papers [11–15] and [16–21], spatial problems for transversely-isotropic elastic and electroelastic bodies respectively were considered. At the same time, analytical solutions of spatial contact problems for transversely isotropic elastic bodies were not obtained when hard punch heated in an arbitrary manner.

In this paper, the problem of thermoelasticity on the indentation of a heated plane hard punch of elliptical cross-section into a transversely isotropic elastic half-space is considered. Expressions of contact stresses and displacements of a heated flat elliptical punch are found explicitly. In the form of inequality, the relationship between the values of the indentation force, the heating temperature, and the thermoelastic properties of a transversely isotropic material is obtained, which makes it possible to predict the appearance of a material separation zone under a flat elliptical punch (for given force and temperature influences). The influence of material properties, heating temperature, and indentation force on the distribution of contact pressure is investigated. It is shown that the appearance of separation (peeling) of the material significantly affects the type of the distribution of contact stresses under the punch.

Formulation of the problem. Let us consider a transversely isotropic half-space that occupies a region $z \le 0$ and into which a heated flat hard punch of elliptical section is pressed without friction. We assume that the axis $0z$ coincides with the axis of symmetry of the transversely isotropic material. The boundary conditions on the surface of the half-space have the following form:

$$\sigma_{xz} = \sigma_{yz} = 0\,,\; z = 0\,;\; \sigma_{zz} = 0\,,\; (x, y) \notin \Omega\,;$$

$$T(x, y, 0) = T_0(x, y), (x, y) \in \Omega; T_0(x, y)_{|\partial\Omega} = 0;$$

$$T(x, y, 0) = 0, (x, y) \in R^2 \setminus \Omega;$$

$$u_z(x, y, 0) = \delta\,,\; (x, y) \in \Omega\,, \tag{1}$$

where $\Omega : x^2/a^2 + y^2/b^2 \le 1$; $T(x, y, 0) > 0$ — punch heating temperature; δ — unknown displacement value. The indentation force applied in the center of the punch is related to the contact pressure by the ratio $P = \iint\limits_{\Omega} p(x, y)\,dxdy$, where $p(x, y)$ is the unknown contact pressure.

Basic relations. The equations of stationary thermoelasticity for an elastic transversely isotropic medium in the absence of body forces and heat sources in the body according to [8] can be written as

$$c_{11}\frac{\partial^2 u_x}{\partial x^2} + \frac{1}{2}(c_{11} - c_{12})\frac{\partial^2 u_x}{\partial y^2} + c_{44}\frac{\partial^2 u_x}{\partial z^2} +$$

$$+\frac{\partial}{\partial x}\left[\frac{1}{2}(c_{11} + c_{12})\frac{\partial u_y}{\partial y} + (c_{13} + c_{44})\frac{\partial u_z}{\partial z}\right] = \beta\frac{\partial T}{\partial x}\,;$$

$$\frac{1}{2}(c_{11} - c_{12})\frac{\partial^2 u_y}{\partial x^2} + c_{11}\frac{\partial^2 u_y}{\partial y^2} + c_{44}\frac{\partial^2 u_y}{\partial z^2} +$$

$$+\frac{\partial}{\partial y}\left[\frac{1}{2}(c_{11} + c_{12})\frac{\partial u_x}{\partial x} + (c_{13} + c_{44})\frac{\partial u_z}{\partial z}\right] = \beta\frac{\partial T}{\partial y}\,;$$

$$c_{44}\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial x^2}\right) + c_{33}\frac{\partial^2 u_z}{\partial z^2} + (c_{13} + c_{44})\frac{\partial}{\partial z}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) = \beta_1\frac{\partial T}{\partial z}\,; \tag{2}$$

$$\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + n_4 \partial^2 T/\partial z^2 = 0\,.$$

In the above expressions, c_{ij} are elastic constants; β, β_1, n_4 — constants depending on the thermophysical properties (thermal conductivity and thermal linear expansion coefficients) of the material. The solution of the system of equations (2) can be represented by means of four potential functions Φ_i (i = 1, 2, 3, 4) in according to [8] in this way:

$$u_x = \partial\Phi_1/\partial x + \partial\Phi_2/\partial x + \partial\Phi_3/\partial y + \partial\Phi_4/\partial x;$$
$$u_y = \partial\Phi_1/\partial y + \partial\Phi_2/\partial y - \partial\Phi_3/\partial x + \partial\Phi_4/\partial y; \qquad (3)$$
$$u_z = m_1\partial\Phi_1/\partial z + m_2\partial\Phi_2/\partial z + m_4\partial\Phi_4/\partial z,$$

where Φ_1, Φ_2, Φ_3 are functions satisfying the equations

$$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + n_j\partial^2/\partial z^2)\Phi_j = 0,$$

also $n_3 = 2c_{44}/(c_{11} - c_{12})$; n_1, n_2 are the roots of the quadratic equation

$$c_{11}c_{44}n^2 - [c_{44}^2 + c_{33}c_{11} - (c_{13} + c_{44})^2]n + c_{33}c_{44} = 0; \qquad (4)$$

$$m_j = \frac{c_{11}n_j - c_{44}}{c_{13} + c_{44}} = \frac{n_j(c_{13} + c_{44})}{c_{33} - n_j c_{44}} \quad (j = 1,2).$$

The function Φ_4 simultaneously satisfies two equations

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + n_4\frac{\partial^2}{\partial z^2}\right)\Phi_4 = 0; \quad \frac{\partial^2\Phi_4}{\partial z^2} = m_3 T.$$

We use the notation $z_j = zn_j^{-1/2}$ $(j = 1,2,3,4)$. Functions $\Phi_1(x, y, z_1)$, $\Phi_2(x, y, z_2)$, $\Phi_3(x, y, z_3)$, $\Phi_4(x, y, z_4)$ will be harmonic functions in the corresponding coordinate system. The constants m_3, m_4 included in relations (3) depend on the elastic and thermophysical properties of a transversely isotropic medium and are written as follows:

$$m_3 = \frac{\beta}{c_{44} + (c_{13} + c_{44})m_4 - c_{11}n_4}; \quad m_4 = \frac{\beta_1(c_{44} - n_4c_{11}) + \beta n_4(c_{13} + c_{44})}{\beta(c_{33} - n_4c_{44}) - \beta_1(c_{13} + c_{44})}.$$

Solution method. We write the temperature field in the form of the harmonic potential of the double layer

$$T(x, y, z_4) = \frac{\partial}{\partial z_4}\left(-\frac{1}{2\pi}\iint_\Omega \frac{T_0(\xi, \eta)d\xi d\eta}{\sqrt{(\xi - x)^2 + (\eta - y)^2 + z_4^2}}\right).$$

It follows from the properties of the derivative of the potential of a simple layer [3] that

$$T(x, y, z_4)|_{z_4 = z = 0} = \begin{cases} T_0(x, y), (x, y) \in \Omega; \\ 0, (x, y) \notin \Omega. \end{cases}$$

Harmonic function is a solution to the Dirichlet problem for the stationary heat conduction equation for a half-space (for a given distribution of the temperature field inside a flat region and zero temperature outside this region on the surface of the half-space [3]). Note that contact problems of thermoelasticity with a known temperature distribution in the contact area and the absence of a temperature field outside it (in the contact plane) were also considered in [8, 24].

Next, we present the solution of problem in the form of a superposition of states, for the first of which we take the function Φ_4 in the form of one of the Boussinesq potentials [2]

$$\Phi_4^{(1)}(x,y,z_4)=F(x,y,z_4)=$$

$$=-\frac{1}{2\pi}\iint\limits_{\Omega}T_0(\xi,\eta)\ln\left(\sqrt{(\xi-x)^2+(\eta-y)^2+z_4^2}+z_4\right)d\xi d\eta\,.$$

For the first state, we also set

$$\Phi_i^{(1)}(x,y,z_i)=\alpha_i F(x,y,z_i)\ \ (i=1,2)\,;\ \Phi_3^{(1)}=0\,,$$

where α_1, α_2 are the unknown constants.

Constants a_1, a_2, we define by means of this way:

$$a_1=-\frac{n_1^{1/2}}{n_4^{1/2}}\frac{(m_2-m_4)}{(m_2-m_1)}\,;\ a_2=\frac{n_2^{1/2}}{n_4^{1/2}}\frac{(m_1-m_4)}{(m_2-m_1)}\,.$$

As a result, for the first state we get

$$\sigma_{xz}^{(1)}=\sigma_{yz}^{(1)}=0\,,\ z=0\,;$$

$$u_z^{(1)}\,|_{z=0}=0\,,\ (x,y)\in\Omega\,;$$

$$\sigma_{zz}^{(1)}\,|_{z=0}=\begin{cases}-\gamma^{\text{Trans}}\,T_0(x,y) & (x,y)\in\Omega;\\ 0, & (x,y)\notin\Omega.\end{cases}$$

We find the value γ^{Trans} in the form

$$\gamma^{\text{Trans}}=\beta_1-m_3(c_{33}m_4-n_4c_{13})+$$

$$+c_{44}m_3n_4^{1/2}\left[\frac{(m_1-m_4)}{(m_1-m_2)}(1+m_2)n_2^{1/2}-\frac{(m_2-m_4)}{(m_1-m_2)}(1+m_1)n_1^{1/2}\right];\qquad(5)$$

$$n_4=k_1/k\,;\ \beta=(c_{11}+c_{12})\alpha+c_{13}\alpha_1\,;\ \beta_1=2c_{13}\alpha+c_{33}\alpha_1\,,$$

where k_1/k is the ratio of the coefficient of thermal conductivity in the direction $0z$ to the coefficient of thermal conductivity in the direction $0x$ (or $0y$); α,α_1 are the coefficients of linear thermal expansion of the material in the direction of $0x$ (or $0y$) and $0z$. In the transition from a transversely isotropic material to the isotropic material, we obtain $\gamma^{\text{Trans}}\to\mu\alpha(1+\nu)/(1-\nu)$ (ν is the Poisson's ratio, μ is the shear modulus), which fully corresponds to the result [5] for an isotropic material. Note that expression (5) for γ^{Trans} it was also used in the papers [22, 23] to find the thermo-stressed state of a transversely isotropic material with an elliptical crack.

For the second state of superposition, we choose the functions Φ_j $(j=1,2,3,4)$ as follows:

$$\Phi_1^{(2)}(x,y,z_1)=\frac{1}{2\pi}\frac{-n_1^{1/2}}{(1+m_1)c_{44}(n_1^{1/2}-n_2^{1/2})}\iint_\Omega p(\xi,\eta)\ln(\rho_1+z_1)\,d\xi\,d\eta\,;$$

$$\Phi_2^{(2)}(x,y,z_2)=\frac{1}{2\pi}\frac{n_2^{1/2}}{(1+m_2)c_{44}(n_1^{1/2}-n_2^{1/2})}\iint_\Omega p(\xi,\eta)\ln(\rho_2+z_2)\,d\xi\,d\eta\,,$$

where

$$\rho_i=\sqrt{(x-\xi)^2+(y-\eta)^2+z_i^2}\quad (j=1,2)\,.$$

In this case we obtain

$$\frac{\partial}{\partial z_i}\Phi_i^{(2)}(x,y,z_i)=\frac{1}{2\pi}\iint_\Omega\frac{p(\xi,y')\,d\,\xi\,d\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2+z_i^2}}\quad (j=1,2)\,.$$

Take also $\Phi_4^{(2)}=\Phi_3^{(2)}=0$.

As a result of superposition of states, we obtain

$$\sigma_{xz}^{(1)}+\sigma_{xz}^{(2)}=\sigma_{yz}^{(1)}+\sigma_{yz}^{(2)}=0\ \text{ for }\ z=0\,;$$

$$(u_z^{(1)}+u_z^{(2)})\,|_{z=0}=\frac{1}{2\pi}A^{\text{Trans}}\iint_\Omega\frac{p(\xi,\eta)\,d\xi\,d\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\,;$$

$$(\sigma_{zz}^{(1)}+\sigma_{zz}^{(2)})\,|_{z=0}=\begin{cases}-p(x,y)-\gamma^{\text{Trans}}T_0(x,y), & (x,y)\in\Omega;\\ 0, & (x,y)\notin\Omega.\end{cases}$$

The value A^{Trans} is determined as

$$A^{\text{Trans}}=\frac{1}{c_{44}(n_1^{1/2}-n_2^{1/2})}\left[\frac{m_2-m_1}{(1+m_1)(1+m_2)}\right]=$$

$$=\frac{c_{11}}{c_{44}}\frac{(n_1^{1/2}+n_2^{1/2})(c_{13}+c_{44})}{(c_{11}n_1+c_{13})(c_{11}n_2+c_{13})}\,. \tag{6}$$

From the obtained expression (6) for a transversely isotropic material, one can easily obtain the case of an isotropic material. Let's put

$$n_1=n_2=1\,;\ c_{11}=\lambda+2\mu\,;\ c_{13}=\lambda\,;\ c_{44}=\mu\,.$$

Then from formula (6) it follows

$$A^{\text{Trans}}\to\frac{\lambda+2\mu}{2\mu(\lambda+\mu)}=\frac{1-\nu}{\mu}\,.$$

Thus, for an isotropic material we obtain a coincidence of the results with the known data [5].

Note that expressions A^{Trans} (6) can be converted to a more convenient form. Using Vieta's theorem for the roots of the quadratic equation, from (6) we obtain

$$A^{\text{Trans}}=\frac{\sqrt{c_{11}}}{(c_{11}c_{33}-c_{13}^2)\sqrt{c_{44}}}\left[\sqrt{c_{11}c_{33}-c_{13}^2-2c_{44}c_{13}+2c_{44}\sqrt{c_{11}c_{33}}}\right].$$

The obtained expression allows to find the desired value by directly substituting the elastic constants of the material into it without first determining the roots of the quadratic equation (4), as in the case of formulas (6).

Correspondence between solutions of contact problems for isotropic and transversely isotropic elastic half-spaces (in contact with heated plane rigid punch of elliptical section). According to the results of [5], the solution of the contact problem of thermoelasticity for an isotropic elastic half-space with boundary conditions (1) can lead to a search for an unknown potential density of a simple layer. It remains to satisfy the boundary condition

$$u_z(x,y,0)=\delta=\frac{(1-\nu)}{2\pi\mu}\iint_{\Omega}\frac{p(\xi,\eta)\,d\xi\,d\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\,. \tag{7}$$

Stress distribution has such form under the plane punch

$$\sigma_{zz}\,|_{z=0}=-p(x,y)-\frac{(1+\nu)}{(1-\nu)}\alpha\,\mu\,T_0(x,y)\,,\ (x,y)\in\Omega\,. \tag{8}$$

As a result of a superposition of states for a transversely isotropic half-space, we obtain

$$u_z(x,y,0)=\delta=\frac{1}{2\pi}A^{\text{Trans}}\iint_{\Omega}\frac{p(\xi,\eta)\,d\xi\,d\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\,. \tag{9}$$

The normal stresses under the plane punch in this case have the form

$$\sigma_{zz}\,|_{z=0}=-p(x,y)-\gamma^{\text{Trans}}\,T_0(x,y)\,,\ (x,y)\in\Omega\,. \tag{10}$$

All other boundary conditions (1) are satisfied. Comparing expressions (7), (8) and (9), (10), we conclude that such contact characteristics as contact pressure and displacement under the plane punch for a transversely isotropic half-space can be calculated from the corresponding expressions for an isotropic half-space by replacing the values $(1-\nu)/\mu$ with A^{Trans} and $(1+\nu)\alpha\mu/(1-\nu)$ by γ^{Trans}.

Solutions of new contact problems. When pressing a flat elliptical punch (in the absence of the rotations around the axes 0*x* and 0*y*) according to the found correspondence of expressions (7), (8) and (9), (10) and results [4, 5] we obtain the values of contact pressure and displacement under the plane punch:

$$p(x,y)=\frac{P-Q_1}{2\pi ab}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{-1/2}+\gamma^{\text{Trans}}\,T_0(x,y)\,;$$

$$\delta=\frac{P-Q_1}{2\pi a}A^{\text{Trans}}\,K(e)\,, \tag{11}$$

where a is the semimajor axis of the ellipse, e is its eccentricity;

$$P\ge Q_1=\gamma^{\text{Trans}}\iint_{Э}T_0(x,y)\,dxdy\,. \tag{12}$$

When inequality (12) is fulfilled, the contact stress under the flat punch are compressive and have a root singularity when approaching the punch boundary, which is determined by the first term in formulas (11).

Consider the distribution of the temperature field under the plane punch in the form

$$T_0(x,y) = T_q(1 - x^2/a^2 - y^2/b^2)^q, \ q > 0 .$$

Then, for a heated plane hard elliptical punch, the values of stresses and displacements under the punch we obtain in the form

$$-\sigma_{zz}(x,y) = \frac{P - Q_1}{2\pi ab}(1 - x^2/a^2 - y^2/b^2)^{-1/2} + \gamma^{\text{Trans}} T_q (1 - x^2/a^2 - y^2/b^2)^q;$$

$$\delta = \frac{P - Q_1}{2\pi a} B_1^{\text{Piezo}} K(e) ;$$

$$Q_1 = \gamma^{\text{Trans}} \iint_\Omega T_q (1 - x^2/a^2 - y^2/b^2)^q \, dx\, dy \ ; \ \ P \geq Q_1 . \tag{13}$$

Note that formulas (13) have the following physical meaning. When the inequality $P \geq Q_1$ is fulfilled, the plane punch is pressed to the material over the entire contact area and under it there is no area of separation (delamination) of the material. After integration, we obtain the inequality

$$P \geq Q_1 = \frac{1}{q+1} T_q \gamma^{\text{Trans}} \pi ab . \tag{14}$$

If the opposite inequality holds

$$P < \frac{1}{q+1} T_q \gamma^{\text{Trans}} \pi ab$$

tensile stresses arise when apprsoaching the punch edge (due to the first term in the stress components in formulas (13)), i.e. a material separation zone appears.

In [7], for the problem of the contact of a heated plane circular punch with an elastic isotropic half-space, it was proposed to search for a new contact zone, which is smaller than the size of the punch itself, from the problem for a non-planar punch, directing $R \to \infty$. The two-dimensional contact problem of thermoelasticity was considered in a similar way in [24]. Using this approach, and considering for this the problem of a heated paraboloidal punch of elliptical cross section, which is pressed into an elastic transversely isotropic half-space without friction, we obtain

$$\frac{a^3}{R_1} = [(P - Q_1)\ A^{\text{Trans}}]^{1/3} \left[\frac{3}{2\pi} \left(\frac{K(e) - E(e)}{e^2} \right) \right]^{1/3},$$

where e is the eccentricity of the elliptical base of the punch. Directing $R_1 \to \infty$, we get

$$P = Q_1 = \frac{1}{q+1} \gamma^{\text{Trans}} T_q \, \pi a_* b_* ,$$

where a_*, b_* are the semi-axes of the new contact area under the punch. They are smaller than the corresponding semi-axes of the plane punch. However, contact zone remains elliptical and the relation remains $b_*/a_* = b/a$ Using the expression (14) we find further

$$a_* = \sqrt{\frac{P(q+1)}{\gamma^{\text{Trans}} T_q \, \pi \sqrt{1 - e^2}}} \ ; \ b_* = a_* \sqrt{1 - e^2} .$$

Contact stress under a heated plane punch in the case of separation of material near the edge of the punch takes the form

$$-\sigma_{zz}(x,y)=\gamma^{\text{Trans}}T_q(1-x^2/a_*^2-y^2/b_*^2)^q\,, \qquad (15)$$

since the singular term disappears in formulas (13) at $P=Q_1$. Therefore, an increase in punch heating when a certain threshold value is exceeded, which depends on the strength P and thermoelastic properties of the transversely isotropic material, leads to the appearance of separation zone of the material under the plane punch.

Note that for a plane circular punch ($e=0$), the radius of the contact area when the material is separated from the punch according to formulas (15) takes the form

$$a_*=\sqrt{\frac{P(q+1)}{\gamma^{\text{Trans}}T_q\,\pi}}\;,\; b_*=a_*\,.$$

Analysis of the results of numerical investigations. Consider the case of the distribution of the temperature field under a plane circular punch of radius a in the form $T_0(1-x^2/a^2-y^2/a^2)^{1/4}$, where $T_0>0$. We investigate next three cases of punch heating: 1) $T_0=\dfrac{1{,}2\,P}{\gamma^{\text{Trans}}\pi a^2}$; 2) $T_0=\dfrac{1{,}3\,P}{\gamma^{\text{Trans}}\pi a^2}$ and 3) $T_0=\dfrac{1{,}5\,P}{\gamma^{\text{Trans}}\pi a^2}$. First, we verify the fulfillment of inequality (14) to find out whether the material is peeling off. As a result, we obtain that for the first case inequality (14) is satisfied, i.e. material separation under the punch does not occur. At the same time, with increasing heating (cases 2 and 3), such a separation of the material takes place.

After simple calculations for the first case of heating, the pressure expression under the punch takes the form

$$-\frac{\sigma_{zz}}{T_0\,\gamma^{\text{Trans}}}=\frac{1}{60}(1-r^2/a^2)^{-1/2}+(1-r^2/a^2)^{1/4}\,.$$

For the second and third cases, the contact pressure has the same expression

$$-\frac{\sigma_{zz}}{T_0\,\gamma^{\text{Trans}}}=(1-r^2/a_*^2)^{1/4}\,.$$

However, the radii of the new contact area for these cases are different. For the second case of heating $a_*=5a/\sqrt{26}$, at the same time for the third case we find that $a_*=a\sqrt{5/6}$.

Fig. 1 shows the change in contact pressure under the punch, while the pressure curve for case 1 (without separating the material under the punch) is shown by line 1, and for the second and third cases – by means of the lines with corresponding numbers. It is seen that with increasing heating of the flat punch, the contact area with the half-space decreases.

Consider a flat elliptical punch with the distribution of the temperature field under the punch in the form $T_0(1-x^2/a^2-y^2/b^2)^{1/4}$ of $T_0>0$. Put $T_0=P/[(\pi ab)\gamma^{\text{Trans}}\beta^*]$. We study cases of punch heating, assuming that it β^*

takes the following values: 1) $\beta^* = 1000$; 2) $\beta^* = 4$; 3) $\beta^* = 2$; 4) $\beta^* = 1$. Since the value T_0 is inversely proportional to the value β^*, in the latter case we get the highest punch heating. For the selected parameter values β^*, the material does not separate under the punch, and the stress expression under the punch takes the form:

$$-\frac{\sigma_{zz}\,\alpha^*}{[P/(\pi ab)]} = \frac{1}{2}\left[\alpha^* - \frac{4}{5}\right](1 - x^2/a^2 - x^2/b^2)^{-1/2} + (1 - x^2/a^2 - x^2/b^2)^{1/4}.$$

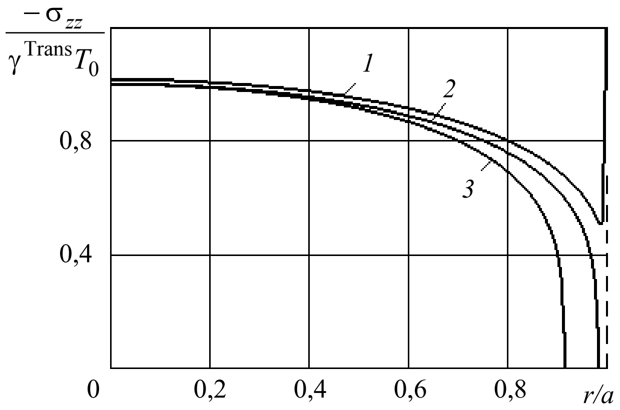

Fig. 1. The stress distribution under a heated flat punch, taking into account the separation of the material

In fig. 2 shows the distribution of the contact pressure under a flat punch with an elliptical section at different values of the heating of the punch.

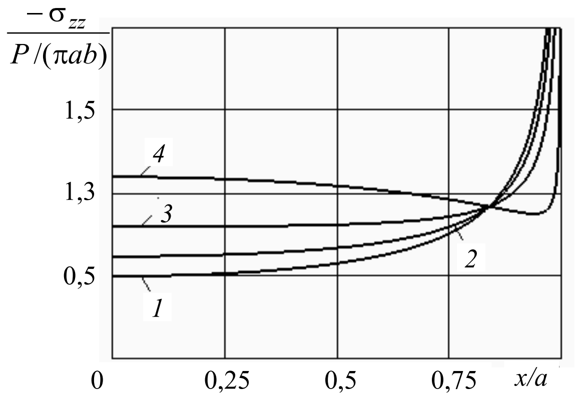

Fig. 2. Stress distribution under a plane elliptical punch

Lines *1–4* in Fig. 2 correspond to punch heating options noted above. It can be seen an increase of the temperature heating of a plane punch lead to the stresses increase in the center of the punch and decrease when approaching its boundary.

Thus, in the paper expressions of contact stresses and displacements of a heated flat elliptical punch are found explicitly. By means of the inequality, the condition for the occurrence of material separation under a flat heated elliptical punch is obtained, which is pressed without friction into the transversely isotropic elastic half-space. This inequality includes the values of the indentation force, temperature heating, and thermoelastic properties of a transversely isotropic material. It is shown that with increasing heating, the region of complete contact (with separation of the material) decreases. The famous results for an elastic isotropic material follow from the obtained data as a special case. The influence of heating on the distribution of contact stresses, as well as the appearance of a region of separation (delamination) of a transversely isotropic material under a punch, is investigated.

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МОДЕЛЮВАННЯ КОНТАКТНОЇ ВЗАЄМОДІЇ НАГРІТОГО ЖОРСТКОГО ЕЛІПТИЧНОГО ШТАМПА З ТРАНСВЕРСАЛЬНО-ІЗОТРОПНИМ ПРУЖНИМ ПІВПРОСТОРОМ / В.С. Кирилюк, О.І. Левчук, В.В. Гавриленко, М.Б. Вітер

Анотація. На основі строгої математичної моделі досліджено задачу контактної взаємодії нагрітого плоского штампа еліптичного перерізу з трансверсально-ізотропним пружним півпростором. Припускається, що поверхня півпростору є площиною ізотропії трансверсально-ізотропного матеріалу, а також має гладкий (без тертя) контакт. У явному вигляді знайдено вирази контактних напружень і переміщення нагрітого плоского еліптичного штампа. У вигляді простої нерівності отримано умову відділення пружного матеріалу від поверхні плоского еліптичного штампа. Виконано числові розрахунки. Вивчено контактну взаємодію нагрітого плоского штампа з урахуванням відділення матеріалу від штампа.

Ключові слова: математична модель, контактна взаємодія, пружний півпростір, трансверсально-ізотропний матеріал, плоский еліптичний штамп, нагрівання, розподіл напружень, ділянка відділення матеріалу.

МОДЕЛИРОВАНИЕ КОНТАКТНОГО ВЗАИМОДЕЙСТВИЯ НАГРЕТОГО ПЛОСКОГО ЖЕСТКОГО ЭЛЛИПТИЧЕСКОГО ШТАМПА С ТРАНСВЕРСАЛЬНО-ИЗОТРОПНЫМ УПРУГИМ ПОЛУПРОСТРАНСТВОМ / В.С. Кирилюк, О.И. Левчук, В.В. Гавриленко, М.Б. Витер

Аннотация. На основе строгой математической модели исследована задача контактного взаимодействия нагретого плоского штампа эллиптического сечения с трансверсально-изотропным упругим полупространством. Предполагается, что поверхность полупространства является плоскостью изотропии трансверсально-изотропного материала, а также имеет гладкий (без трения) контакт. В явном виде найдены выражения контактных напряжений и перемещения нагретого плоского эллиптического штампа. В виде простого неравенства получено условие отделения упругого материала от поверхности плоского эллиптического штампа. Выполнены числовые расчеты. Изучено контактное взаимодействие нагретого плоского штампа с учетом отделения материала от штампа.

Ключевые слова: математическая модель, контактное взаимодействие, упругое полупространство, трансверсально-изотропный материал, плоский эллиптический штамп, нагрев, распределение напряжений, область отделения материала.