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ELECTRIC HEATER MATHEMATICAL MODEL FOR CYBER-PHYSICAL SYSTEMS

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Abstract. The article discusses the heat and mass transfer dynamic model for an electric heater with lumped parameters, which allows transient processes simulation for the main influences. The proposed model is recommended to be used in cyber-physical systems for forecasting and evaluating the effectiveness of control systems integrated into a single information management system. The developed model can be used by specialists for the analysis and synthesis of control systems for balanced ventilation systems or industrial air conditioners. As an example, a numerical simulation of transient processes along the action main channels for an electric heater HE 36/2 manufactured by VTS CLIMA was carried out. The significant advantage of the proposed model is the possibility for using it for the synthesis and analysis of multidimensional control systems.

Keywords: cyber-physical system, digital twin, mathematical model, state space, electric heater.

INTRODUCTION

In the modern world, the managing technical systems tasks are closely intertwined with the social sphere of human activity. The growth and complexity of management situations in technical systems requires the use of new scientific approaches to management. The computer-integrated control systems properties study from the interaction view point of physical and computational component is a priority in the modern science about cyber-physical systems [1]. To date, such systems have not yet received a generally accepted definition, since they lie at the intersection of several human spheres of activities. Their common characteristic is the interaction between computing and physical processes, where the computing system constantly receives data from the environment and uses them to further optimize the physical processes control. Cyber-physical systems include, for example, networked intelligent control of power supply or transport, automated control systems in production and agriculture, and much more.

While designing cyber-physical systems, it is necessary to solve a variety of interrelated problems. One of the important tasks is the development of real object's digital twin based on mathematical modeling [2, 3]. A digital twin is a digital model obtained based on information from sensors installed on a physical object, which allows you to simulate object's behavior in the real world. Fundamentally, a digital twin can be defined as an ever-changing digital profile containing historical and most current data about a physical object or a production process. The digital twin is created to predict the real object's behavior in the "virtual space" and to adopt a control strategy. Sometimes the time required to simulate a mathematical model can exceed the physical process flow, and the calculated data becomes outdated. For this reason, the adequate mathematical models' development for technical systems that can be used for calculations in "real time" is an urgent task.

The heat power industry plays an important role in modern society as a tool for creating material wealth and a means of servicing human life. The one of main units in heat power engineering is a heat exchanger, which takes on "various forms" in a particular human activity area. Heat exchangers include heaters, which are designed to heat the air due to the heat flow from the primary heat transfer fluid. In industry, air heaters are often used to heat air in heating, ventilation, air conditioning systems, various dryers and other technical systems [4].

In the mining and coal industries, the most important underground mining task is to organize the effective ventilation [5]. This problem has both technical and economic aspects. In technical terms, the key issue in organizing the mine ventilation is heating the air supplied to the mine to the temperature above 0 °C (from +2 °C to +5 °C). For many decades, the main technical solution for heating the air supplied to the mine has been provided by the water heaters use. Despite the design simplicity, using water heaters creates a variety of problems associated with complex installation, water treatment, continuous maintenance, the danger of water freezing in heaters and, as a consequence, the need to maintain the supplied to the mine air temperature above the optimum level.

Recently, the industry began to use air heating systems, which have established themselves as fast-acting systems with low specific capital investments. For air heating of shopping and business centers, warehouses and industrial buildings, centralized ventilation and air conditioning systems are used, in which air heaters are the main equipment [6]. Residential air heating systems use low-power electric heaters with a distributed automatic control system [7].

Considering the above, in recent years, electric heaters of high power (up to 30 kW) are increasingly used for heating air in various industries. Electricity transportation has many advantages over other types of heat carriers, and the use of thermoelectric heaters (TEH) can significantly increase the air heaters efficiency and design air heaters with a wide heat output range. In addition, electric heaters are efficient at outdoor temperatures down to –50 °C and do not lose their functionality in the event of an emergency power outage.

RESEARCH PROBLEM STATEMENT

The aim of this publication is to develop the heat transfer process dynamic model for an electric air heater, which can be used as a digital twin for a cyber-physical system. An additional requirement for the mathematical model is its ease of use by existing modeling tools on a personal computer.

HEAT TRANSFER PROCESS MATHEMATICAL MODELING

Heat transfer processes are discussed in many publications. To simulate the dynamic processes in heat exchangers, researchers use mathematical models with lumped [8–10] and distributed [11, 12] parameters. Models with lumped parameters provide an analytical solution to the heat transfer task. Models with distributed parameters describe the physical process more accurately, but in the analytical modeling of such processes, transcendental functions appear in the solution [12], which complicates the calculations. In practice, numerical methods are used to solve distributed heat transfer problems.

The existing dynamic models of air heaters [8–12] are based on the mathematical description of the heat exchange process for devices where the primary heat transfer fluid is steam or water, since these devices have long been used in production as part of automatic control systems. For electric heaters, stationary models are known, which are used to design the equipment. Considering that recently, high-power electric heaters have been used in production as part of cyber-physical systems, we will develop the electric heater dynamic model.

ELECTRIC HEATER MATHEMATICAL MODEL

The modern electric heaters designs are based on blowing the air through a bundle of heating elements, while convective heat exchange is carried out between the moving air and the heater surface. Considering surface heat exchangers used in industrial systems, it is necessary to take into account that the air is intensively mixed by fans. Practical research of unsteady heat transfer characterizes the air heating process as clearly aperiodic [11], which is described with sufficient accuracy by second-order transfer functions. Thus, a lumped-parameter mathematical model for a water cooler would be perfectly acceptable.

When developing a mathematical model, the following simplifications were adopted: there is no heat exchange with the environment; the model contains two dynamic elements with lumped parameters – tubular electric heaters and the air volume of heater. Physical properties of material flows and the heat exchange surface are reduced to the average values of device's operating range. The electric heater design diagram is shown in Fig. 1.

To heat the air in the heater, electric heaters are used, which are connected to the mains through a triac electrical converter. The electrical converter in proportion to the regulator signal changes the electrical power $N_E(t)$, due to which the heating elements are heated to the temperature $\theta_E(t)$. All electrical power is used to heat the air. Air is supplied crosswise to the electric heaters location with a flow rate $G_A(t)$. The inlet air temperature is $\theta_{A0}(t)$, the outlet air temperature is $\theta_A(t)$. The heater geometrical dimensions L, C, H are the depth, width and height, respectively. Let us analyze the heat balance in the dynamics for each dynamic element of the air heater.

The heat balance for heating elements is:

$$N_E - \alpha_0 F_0(\theta_E - \theta_A) = M_E c_E \frac{d\theta_E}{dt} , \qquad (1)$$

where c_E is the heat capacity of heating elements (heating elements contain metal and dielectric parts, so their heat capacity is averaged in proportion to the mass fraction of each part); M_E is the total mass of heating elements; α_0 is the heat transfer coefficient between the air in the heater and the heating elements outer surface; F_0 is the heating elements surface ribbing total area. Let us present equation (1) in canonical form:

$$T_E \frac{d\,\Delta\theta_E}{dt} + \Delta\theta_E = k_0\,\Delta N_E + k_1\,\Delta\theta_A , \qquad (2)$$

where $K_E = \alpha_0 F_0$; $T_E = \dfrac{c_E M_E}{K_E}$; $k_0 = \dfrac{1}{K_E}$; $k_1 = 1$.

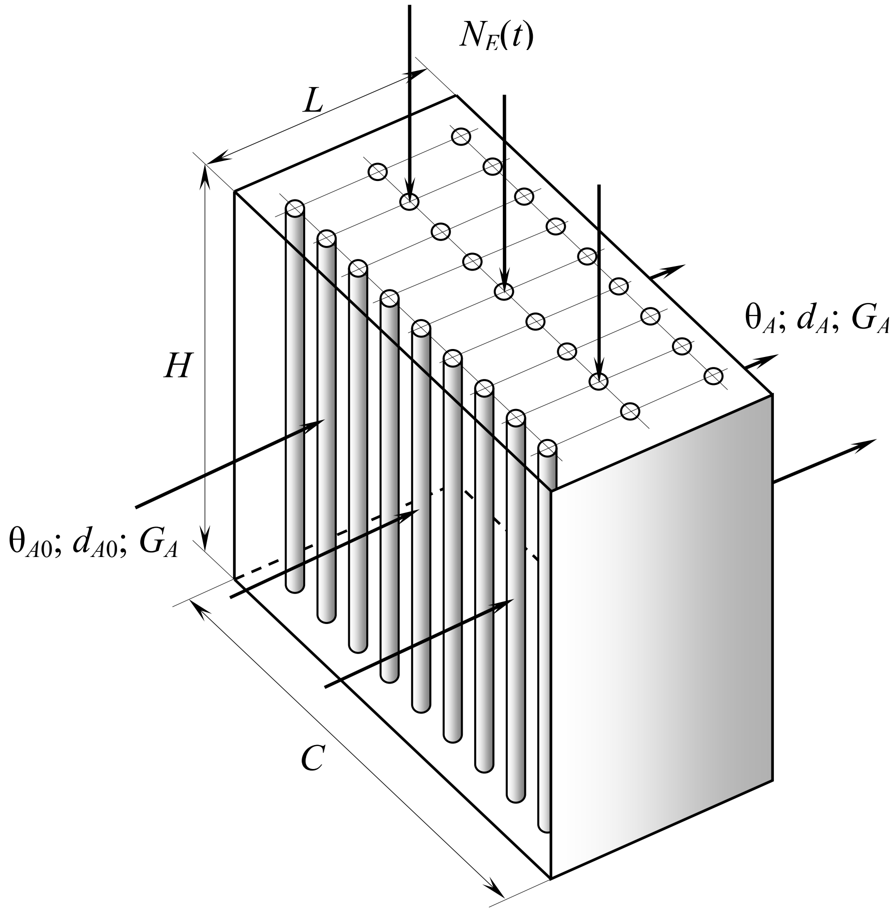

Fig. 1. Design diagram for modelling an electric air heater

The heat balance for the air heater air space is:

$$G_A c_A(\theta_{A0} - \theta_A) + \alpha_0 F_0(\theta_E - \theta_A) = M_A c_A \frac{d\theta_A}{dt} , \tag{3}$$

where c_A is the air heat capacity; M_A is the air mass in the heater volume $L \times C \times H$. After linearization, let us transform equations (3) to the canonical form:

$$T_A \frac{d\,\Delta\theta_A}{dt} + \Delta\theta_A = k_2\,\Delta\theta_E + k_3\,\Delta\theta_{A0} + k_4\,\Delta G_A , \tag{4}$$

where $K_A = c_A G_A + \alpha_0 F_0$; $T_A = \dfrac{c_A M_A}{K_A}$; $k_2 = \dfrac{\alpha_0 F_0}{K_A}$; $k_3 = 1 - k_2$; $k_4 = \dfrac{c_A(\theta_{A0} - \theta_A)}{K_A}$.

Electric heaters are often used in air conditioning systems. For such systems, in addition to the temperature regime, it is also important to simulate the heated air humidity characteristics. It is known from the physical properties of air that with an increase in the vapor-air mixture temperature, its relative humidity decreases (an increase in temperature by 1 °C leads to a decrease in moisture by about 3%), while the air moisture content remains constant [6]. In addition, atmospheric pressure affects humidity. For these reasons, we write the material balance for the air space of the heater in air moisture content terms:

$$\frac{G_A}{1000}(d_{A0} - d_A) = V_A \frac{d\rho_A}{dt}, \tag{5}$$

where $d_{A0}(t)$ and $d_A(t)$ are the air moisture content at the air inlet and outlet, respectively; ρ_A is the air density; V_A is the heater air space volume. The humid air density is determined from the equation [13]:

$$\rho_A = \omega\left(1 + \frac{d_A}{1000}\right), \tag{6}$$

where ω is the dry air density under normal conditions. Taking into account (6), equation (5) after mathematical transformations will take the form:

$$T_d \frac{d\,\Delta d_A}{dt} + \Delta d_A = k_5\,\Delta d_{A0} + k_6\,\Delta G_A, \tag{7}$$

where $T_d = \frac{\omega V_A}{G_A}$; $k_5 = 1$; $k_6 = \frac{d_{A0} - d_A}{G_A}$.

Equations (2), (4) and (7) represent a dynamic model of heat and mass transfer processes when humid air is heated by an electric heater:

$$\begin{cases} T_E \frac{d\,\Delta\theta_E}{dt} + \Delta\theta_E = k_0\,\Delta N_E + k_1\,\Delta\theta_A; \\ T_A \frac{d\,\Delta\theta_A}{dt} + \Delta\theta_A = k_2\,\Delta\theta_E + k_3\,\Delta\theta_{A0} + k_4\,\Delta G_A; \\ T_d \frac{d\,\Delta d_A}{dt} + \Delta d_A = k_5\,\Delta d_{A0} + k_6\,\Delta G_A. \end{cases} \tag{8}$$

Mathematical model (8) in the state space has the form:

$$\mathbf{X}' = \mathbf{AX} + \mathbf{BU}, \tag{9}$$

where $\mathbf{X}' = \begin{bmatrix} \Delta\theta'_A \\ \Delta d'_A \\ \Delta\theta'_E \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -1/T_A & 0 & k_2/T_A \\ 0 & -1/T_d & 0 \\ k_1/T_E & 0 & -1/T_E \end{bmatrix}$; $\mathbf{X} = \begin{bmatrix} \Delta\theta_A \\ \Delta d_A \\ \Delta\theta_E \end{bmatrix}$;

$$\mathbf{B} = \begin{bmatrix} k_3/T_A & 0 & k_4/T_A & 0 \\ 0 & k_5/T_d & k_6/T_d & 0 \\ 0 & 0 & 0 & k_0/T_E \end{bmatrix}; \quad \mathbf{U} = \begin{bmatrix} \Delta\theta_{A0} \\ \Delta d_{A0} \\ \Delta G_A \\ \Delta N_E \end{bmatrix}., \tag{10}$$

Let us solve the system of differential equations (8) with respect to the sought variables using the Laplace transform:

$$\begin{cases} \Delta\theta_E(T_E p + 1) = k_0\,\Delta N_E + k_1\,\Delta\theta_A; & (11) \\ \Delta\theta_A(T_A p + 1) = k_2\,\Delta\theta_E + k_3\,\Delta\theta_{A0} + k_4\,\Delta G_A; & (12) \\ \Delta d_A(T_d p + 1) = k_5\,\Delta d_{A0} + k_6\,\Delta G_A. & (13) \end{cases}$$

From (11) we find $\Delta\theta_E(p)$, then we substitute it in (12), and after grouping similar ones we obtain:

$$\Delta\theta_A = \frac{1}{a_2 p^2 + a_1 p + 1}[(b_1 p + b_0)\Delta\theta_{A0} + (b_3 p + b_2)\Delta G_A + b_4 \Delta N_E], \quad (14)$$

where

$$a_2 = \frac{T_E T_A}{1 - k_1 k_2}; \; a_1 = \frac{T_E + T_A}{1 - k_1 k_2}; \; b_0 = \frac{k_3}{1 - k_1 k_2}; \; b_1 = \frac{k_3 T_E}{1 - k_1 k_2};$$

$$b_2 = \frac{k_4}{1 - k_1 k_2}; \; b_3 = \frac{k_4 T_E}{1 - k_1 k_2}; \; b_4 = \frac{k_0 k_2}{1 - k_1 k_2}.$$

From (13) we obtain a solution for the variable Δd_A:

$$\Delta d_A = \frac{1}{T_d p + 1}[k_6 \Delta d_{A0} + k_7 \Delta G_A].' \quad (15)$$

Applying the inverse Laplace transform, it is possible to find an analytical solution for (14) and (15) by the influence channels. We represent (14) and (15) by a multidimensional model in the Laplace domain

$$\mathbf{Y} = \mathbf{W}\mathbf{Z}, \quad (16)$$

where

$$\mathbf{Y} = \begin{bmatrix} \Delta\theta_A \\ \Delta d_A \end{bmatrix}; \; \mathbf{W} = \begin{bmatrix} W_{11} & 0 & W_{13} & W_{14} \\ 0 & W_{22} & W_{23} & 0 \end{bmatrix}; \quad (17)$$

$$\mathbf{Z}^{\mathbf{T}} = [\Delta\theta_{A0} \; \Delta d_{A0} \; \Delta G_A \; \Delta N_E];$$

$$W_{11} = \frac{b_1 p + b_0}{a_2 p^2 + a_1 p + 1}; \; W_{13} = \frac{b_3 p + b_2}{a_2 p^2 + a_1 p + 1};$$

$$W_{14} = \frac{b_4}{a_2 p^2 + a_1 p + 1}; \; W_{22} = \frac{k_6}{T_d p + 1}; \; W_{23} = \frac{k_7}{T_d p + 1}.$$

The proposed electric heater dynamic model in the differential equations system form (8), in the state space (9), or in the Laplace region (16)–(17), when modelling mass transfer processes, uses the air moisture content, which by definition is expressed as:

$$d_A = \frac{M_W}{M_A}, \quad (18)$$

where M_W is the water steam mass in the air mixture; M_A is the dry air mass in the air mixture. Often, when synthesizing and analysing industrial air conditioners control systems, it is necessary to use the relative humidity of air, which is defined as:

$$\varphi_A = \frac{P_P}{P_S}, \quad (19)$$

where P_S is the saturated steam pressure; P_P is the water steam partial pressure. The water steam partial pressure of humid air is uniquely determined by the moisture content and does not depend on the steam temperature [14]

$$P_P = \frac{P_B \, d_A}{623 + d_A}. \tag{20}$$

where P_B is the barometric air pressure. The saturated steam pressure can be determined by the approximating dependence of N.I. Filneem [15]

$$P_S = 133{,}3 \cdot 10^{\frac{156+8{,}12\,\theta_A}{236+\theta_A}}. \tag{21}$$

If the air temperature θ_A and its relative humidity φ are known, using dependencies (18)–(21), it is possible to determine the air mixture moisture content d

$$d_A = \frac{83045{,}9\,\varphi_A\,10^{\frac{156+8{,}12\,\theta_A}{236+\theta_A}}}{P_B - 133{,}3\,\varphi_A\,10^{\frac{156+8{,}12\,\theta_A}{236+\theta_A}}}. \tag{22}$$

Moisture content recalculation into relative humidity can be carried out according to the inverse relationship

$$\varphi_A = \frac{P_B \, d_A}{133{,}3\,(623 + d_A)\,10^{\frac{156+8{,}12\,\theta_A}{236+\theta_A}}}. \tag{23}$$

It is worth noting that the moisture content (22) and relative humidity (23) are calculated in absolute values, and in models (8), (9), and (16), variables increments are considered, and this feature must be taken into account during modelling.

AIR HEATER DYNAMIC MODE SIMULATION

As an example, let us carry out dynamic processes' simulation modelling for the eclectic heater HE 36/2 of the central air conditioner CV-P 2L N-63B/F-N manufactured by VTS CLIMA. In Table the thermal and physical parameters for modelling the air heater HE 36/2 are shown.

Parameters of the electric heater HE 36/2

N	Parameter name	Symbol	Numerical value	Dimension
1	Electric heater dimensions	$H \times C \times L$	$0{,}38 \times 1 \times 0{,}4$	*m*
2	Heating element material density	ρ_E	7900	*kg/m*^{3}
3	Heating element material heat capacity	c_E	460	*J/(kg* ^oC)
4	Heating element material mass	M_E	0,6	*kg*
5	Air flow through the electric heater	G_A	0,43	*kg/s*
6	Dry air density	ω	1,2	*kg/m*^{3}
7	Air heat capacity	c_A	1010	*J/(kg* ^oC)
8	Air mass in the heater	M_A	0,182	*kg*
9	Heat exchange area between heating elements and air	F_0	0,306	m^2
10	Heat transfer coefficient between heating elements and air	α_0	161	$W/(m^{2o}C)$
11	Input air temperature	θ_{A0}	11	^oC
12	Output air temperature	θ_A	15	^oC
13	Input air moisture content	d_{A0}	9	*g/kg*
14	Output air moisture content	d_A	9	*g/kg*
15	Heater power	N_E	3300	*W*

The elements for the matrices **A** and **B** of the electric heater model (9) were calculated according to the dependencies (10) in the MatLab environment using the program module:

```
H=0.38; C=1; L=0.4;
A0=161; F0=0.306; Ga=0.43; w=1.2;
Ma=0.182; ca=1010; Me=0.6; ce=460;
TetA0=11; TetA=15; dA0=9; dA=9;
Ke=A0*F0; Te=ce*Me/Ke; k0=1/Ke; k1=1;
Ka=ca*Ga+A0*F0; Ta=ca*Ma/Ka; k2=A0*F0/Ka; k3=1-k2;
k4=ca*(TetA0-TetA)/Ka;k=1-k1*k2;
Td=w*H*L*C/Ga; k5=1; k6=(dA0-dA)/Ga;
A=[-1/Ta,0,k2/Ta;0,-1/Td,0;k1/Te,0,-1/Te];
B=[k3/Ta,0,k4/Ta,0;0,k5/Td,k6/Td,0;0,0,0,k0/Te];
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According to the calculations, the following numerical values were obtained for the matrices:

$$\mathbf{A}=\begin{bmatrix}-2{,}631 & 0 & 0{,}268\\ 0 & -2{,}357 & 0\\ 0{,}179 & 0 & -0{,}178\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}2{,}362 & 0 & -21{,}98 & 0\\ 0 & 2{,}358 & 0 & 0\\ 0 & 0 & 0 & 0{,}0036\end{bmatrix}.$$

The dynamic mode simulation for the HE 36/2 electric heater was carried out in the Simulink MatLab environment using the State Space function block. Fig. 2, *a–c* show the research results for transient processes in the electric heater for the perturbation channels, and Fig. 2, *d* – shows such results for the control channel.

From the conducted research, it can be concluded that a change in the inlet temperature $\Delta\theta_{A0}$ does not affect the air moisture content at the electric heater outlet Δd_A (Fig. 2, *a*). A change in the air moisture content at the inlet Δd_{A0} does not affect the outlet temperature (Fig. 2, *b*). A change in the steam-air mixture flow rate ΔG_{A0} affects the heated air temperature $\Delta\theta_A$ (Fig. 2, *c*). Control actions ΔN_E do not affect the heated air moisture content Δd_A (Fig. 2, *d*). The considered effect of heating the steam-air mixture on air heaters is recommended to be used in the artificial microclimate control systems development. The air moisture content use as a control variable (relative humidity instead) can significantly reduce the mutual influences of the temperature and humidity control loops in the air conditioner and improve the control system dynamic properties.

The resulting transients are aperiodic without delay. The control channel inertia is greater than the disturbance channels inertia. This dynamic is explained by the small heater airspace volume V_A=0,152 m^3 with a relatively large flow rate of heated air G_A=0,43 kg/s. For this reason, if necessary, it is recommended to calculate the relative humidity at the output of the electric heater according to dependence (23). The transient processes inertia in the electric heater is comparable to the temperature sensor inertia, therefore, the dynamic sensor properties must be taken into account when developing a control system. From practical recommendations in control systems with electric heaters, it is sufficient to use control PI-law. Using the more complex control law is not justified for these devices.

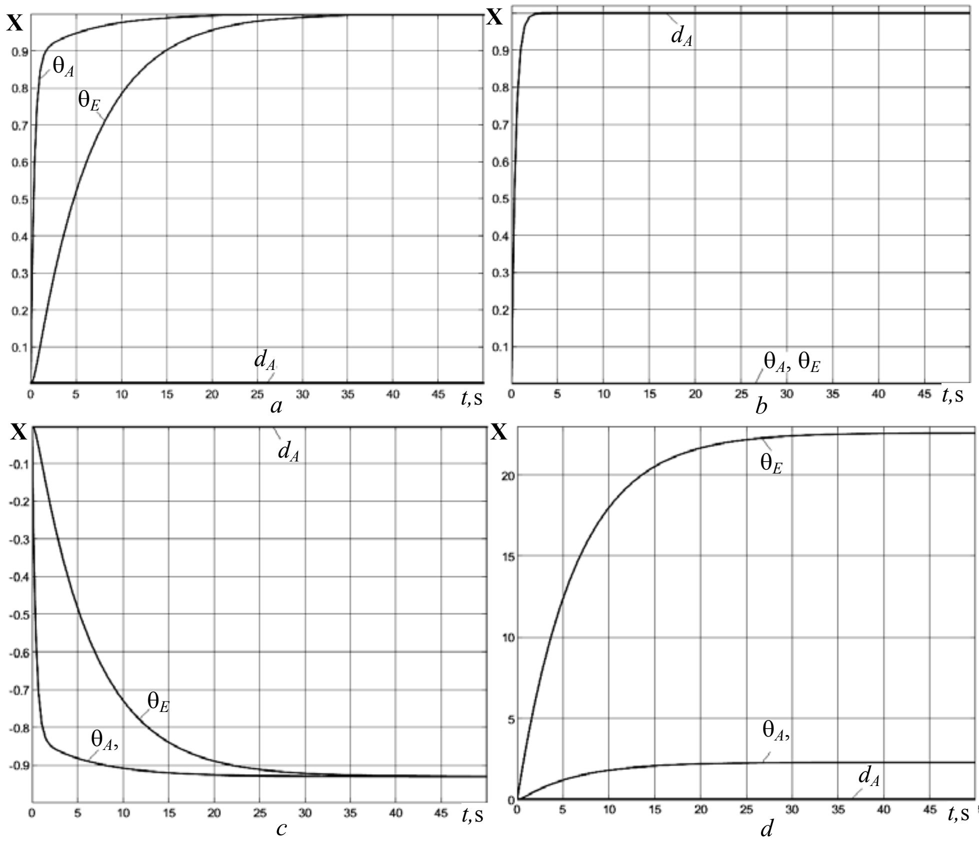

Fig. 2. Transient processes modelling in the heater HE 36/2 for the action main channels: *a* — $\Delta\theta_{A0} \to \mathbf{X}$, $\Delta\theta_{A0} = 1\,^{o}C$; *b* — $\Delta d_{A0} \to \mathbf{X}$, $\Delta d_{A0} = 1$ *g/kg*; *c* — $\Delta G_{A0} \to \mathbf{X}$, $\Delta G_{A0} = 0{,}1$ *kg/s*; *d* — $\Delta N_E \to \mathbf{X}$, $\Delta N_E = 1000\,W$

CONCLUSIONS

The article proposes a mathematical description of heating the steam-air mixture by an electric heater. The mathematical model is obtained in the equivalent dependencies form: the system of differential equations (8); in the state space (9); in the Laplace domain (16). The mathematical model can be used as a digital twin for a digital controller with a reference model. In addition, the proposed model is recommended to be used for the synthesis and analysis of control systems for balanced ventilation systems or industrial artificial microclimate systems. The mathematical description makes it possible to obtain the air heater dynamic characteristics for the main channels of control and disturbance. An example for the transient processes simulation for an electric heater HE 36/2 is given.

The choice of a mathematical model (8), (9) or (16) is determined by the approach to designing a cyber-physical system. The model in the state space (9) has a number of advantages over the notation in the transfer functions form (16). For example, it allows to describe the internal model structures using a minimum number of parameters. An additional argument in favor of vector models is a large number of software packages for the computer analysis of technical systems in the state space, which allow complex mathematical research to be carried out with little time.

A separate direction in the technical systems modeling is the identification of dynamic model coefficients and the resulting model adequacy assessment. The electric heater model coefficients were calculated according to the parameters given in Table. Thermal and physical parameters for the considered model are determined with a high accuracy from reference books, except for the heat transfer coefficient α_0. This coefficient depends on many factors [16]. Therefore, the heat transfer coefficient should be attributed to the proposed model parametric uncertainty, which affects almost all elements in matrices **A** and **B** of model (9). The heat transfer coefficient can be determined with a high accuracy based on experimental studies. Thus, there is a need for the proposed model parametric adaptation to the specific conditions of air preparation on the experiment basis. This research will be the topic for the next publication, where the model coefficients will be identified based on an active experiment using computer technology.

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МАТЕМАТИЧНА МОДЕЛЬ ЕЛЕКТРИЧНОГО КАЛОРИФЕРА ДЛЯ КІБЕРФІЗИЧНИХ СИСТЕМ / Н.Д. Панкратова, І.М. Голінко

Анотація. Розглянуто динамічну модель тепло- і масообміну для електричного калорифера із зосередженими параметрами, яка дозволяє проводити імітаційне моделювання перехідних процесів за основними каналами впливу. Запропоновану модель рекомендується використовувати в кіберфізичних системах для прогнозування та оцінювання ефективності систем управління, інтегрованих в єдину інформаційну систему управління. Розроблену модель можуть використовувати фахівці для аналізу та синтезу систем управління припливно-витяжної вентиляції або промислових кондиціонерів. Як приклад виконано числове моделювання перехідних процесів за основними каналами впливу для електрокалорифера HE 36/2 виробництва VTS CLIMA. Суттєвою перевагою запропонованої моделі є можливість використання її для синтезу та аналізу багатовимірних систем управління.

Ключові слова: кіберфізична система, цифровий двійник, математична модель, простір станів, електричний калорифер.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ЭЛЕКТРИЧЕСКОГО КАЛОРИФЕРА ДЛЯ КИБЕРФИЗИЧЕСКИХ СИСТЕМ / Н.Д. Панкратова, И.М. Голинко

Аннотация. Рассмотрена динамическая модель тепломассообмена для электрического калорифера с сосредоточенными параметрами, которая позволяет проводить имитационное моделирование переходных процессов по основным каналам воздействия. Предложенную модель рекомендуется использовать в киберфизических системах для прогнозирования и оценки эффективности систем регулирования, которые интегрированы в единую информационную систему управления. Разработанную модель могут использовать специалисты для анализа и синтеза систем регулирования приточно-вытяжной вентиляции или промышленных кондиционеров. В качестве примера проведено численное моделирование переходных процессов по основным каналам воздействия для электрического калорифера HE 36/2 производства фирмы VTS CLIMA. Существенным преимуществом предложенной модели является возможность ее использования для синтеза и анализа многомерных систем управления.

Ключевые слова: киберфизическая система, цифровой двойник, математическая модель, пространство состояний, электрический калорифер.