

МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ, ПРОБЛЕМИ І ТЕХНОЛОГІЇ ДОСЛІДЖЕННЯ СКЛАДНИХ СИСТЕМ

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MODELING OF A TEMPERATURE FIELD FOR EXTRUDER BODY

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Abstract. The paper considers the process of induction heating of the extruder body, the temperature of which determines the degree of heating of the polymer mixture in the zone of loading the dry mixture. A mathematical model of this process is formulated taking into account radiant heat transfer in the gap between the inductor and the case. An iterative numerical-analytical method is proposed for solving the corresponding nonlinear boundary value problem of housing heating, at the first iteration of which a linear boundary value problem is solved (without taking into account radiant heat transfer). At the subsequent stages, a nonlinear boundary value problem is solved. The iterative method is based on the application of integral transformations of the linear part of the problem, followed by an iterative scheme for finding a nonlinear problem. This scheme is based on the algorithms for the equivalent simplification of the expressions obtained by solving the problem. The results of mathematical modeling of the corresponding algorithms are presented.

Keywords: equivalent simplification, extruder, induction heating, integral transformations, polymer, Bessel functions.

INTRODUCTION

The processes available in the extruders in the manufacture of products from polyethylene dry mixes are characterized by the versatility of the tasks to be solved. As is known, [1-4], heat and mass transfer processes in extruders are divided into several zones: the loading zone of the dry mixture at the inlet to the extruder, which heats the mixture to a temperature close to the melting point of the polymer, the delay zone in which the formation of wall polymer melt films, melting zone where the mixture is melted, dosing zone in which the polymer melt is cooled, product forming zones.

Analysis of the literature on these processes in the areas of loading, melting, dosing and cooling of the polymer mixture and polymer melts in the manufacture of cable insulation at ultra-high voltages, showed that the process of heating the screw housing in the extruder is not considered. In our opinion, this process significantly affects the temperature of the mixture in the loading zone along with the heating of the mixture due to the dry friction of the mixture.

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FORMULATION OF THE PROBLEM

Consider the process of induction heating of the extruder body to a given temperature. We set the heat transfer conditions by convection and radiation at the upper limit.

The density of internal heat sources is the electromagnetic energy released per unit time per unit volume. Due to the surface effect, the distribution of internal heat sources is significantly heterogeneous and depends on the electrophysical properties of the load, which change during heating.

The whole heating process is divided into intervals, in each of which the loading properties are assumed to be constant:

$$L_i(m) = \{0,9;1,45;2,05;2,65\}$$
.

Fig. 1 [5] shows a graph of the distribution of specific volume over the length of the cylinder.



Fig. 1. Distribution of specific volumetric power by the length of sections 1-4

The graph shows that the first section of the extruder (first inductor) releases maximum power, because in this section it is necessary to heat the polymer from the initial temperature to the melting point, and in the other zones is heated and maintained at a given temperature.

The distribution of the temperature field of the body T_k is described by the equation of thermal conductivity, which in the cylindrical coordinate system has the form

$$\frac{\partial T_k}{\partial t} = a_k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_k}{\partial r} \right) + \frac{\partial^2 T_k}{\partial z^2} \right) + \frac{a_k}{\lambda} w, \tag{1}$$

where $a_k = \lambda_k / (c_k \rho_k)$ is the coefficient of thermal conductivity; λ_k, c_k, ρ_k thermal conductivity, W /(m C), specific heat and density of the housing material, respectively, g(r,z) — the function of the distribution of the density of internal energy sources in the material, W /m³. Given that the depth of penetration of electromagnetic energy from the inductor is small, $\Delta \ll 1$, assume that it acts on the outer surface of the housing $r = r_4$. Taking into account the above, the specific power on the outer surface of the housing is equal to [5]

$$w_0 = p_0 e^{(b/2 - \Delta)/\Delta} = \frac{10^{-4}}{4\pi} \sqrt{10^{-1} f_k \rho'} H_{me}^2 e^{(x - b/2)/\Delta}$$

If the density of the heat source on the surface of the ingot is given, use the first part of formula (2), if you know the magnetic field strength on the surface of the ingot, use the second part of this formula.

The initial and boundary conditions are:

$$T_k \mid_{t=0} = T_0;$$
 $T_k \mid_{r=r_4} = T_{0k}; \left. \frac{\partial T_k}{\partial r} \right|_{r=r_3} = -\frac{q^k}{h_1};$ (1)

$$\left[\frac{\partial T_k}{\partial z} + h_1 T_k\right]\Big|_{z=0} = 0, \left[\frac{\partial T_k}{\partial z} + h_1 T_k\right]\Big|_{z=L} = 0,$$
(2)

where $h_1 = \alpha_k / \lambda_k$, α_k — heat transfer coefficient of the case into the environment. For the contact surface of the steel pipe with air $\alpha_k = 9 \text{ W}/(\text{m}^2 \text{ c})$; T_0 — temperature of surrounded space; T_{0k} — temperature on the bound of inductor.

Boundary condition on the side surface of a cylindrical workpiece:

$$\left[\lambda_1(T_k)\frac{\partial T_k}{\partial r} + \alpha_1(T_k)T_k(r, z, t)\right]_{r=r_4} = L_G + N_G(T_k);$$
(3)

$$L_G = \alpha(T_k)T_0 + \varepsilon_1 \left(\frac{T_0}{100}\right)^4; \ N_G = \varepsilon_1 \left(\frac{T_k(r_4, z, t)}{100}\right)^4; \ \varepsilon_1 = 0,65.$$
(4)

The function $g(r_4, z)$ in this case is the temperature at which the inductor heats the surface of the housing, ie. $g(r_4, z) = T^{\text{ind}}$.

Values of thermophysical parameters for the case (steel) at temperature T = 300 K are equal: $\rho = 7845 \text{ kg} / \text{m}^3$, $c_V = 0.461 \text{ Kt/(kg \cdot c \cdot K)}$, $\lambda = 58$, $\alpha = 10.51/\text{K}$.

SOLUTION METHOD

Since the boundary value problem (1)–(5) is nonlinear, we will perform mathematical modeling according to the iterative scheme [6]. First, we obtain the solution of the problem in a linear approximation. We apply to equation (1) the integral transformation over the variable z:

$$\frac{d^2 Z(\delta_k)}{dz^2} + \delta_k^2 Z(\delta_k) = 0.$$

Custom conversion functions:

$$Z(\delta_k z) = \frac{1}{\|Z_k\|^2} \left[\sin \delta_k z - \frac{\delta_k}{h_1} \cos \delta_k z \right].$$

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The integral of the product of eigenfunctions is equal to:

$$\int_{k=1}^{M} \int_{j=1}^{M} Z_k Z_j dz = \sum_{\pm} \frac{\delta_j}{2p_{kj}} \left[\left(\frac{\delta_k \delta_j}{h_1^2} \mp 1 \right) (1 - \cos p_{kj} L) + \frac{p_{kj}}{h_1} \sin p_{kj} L \right], \quad p_{kj} = \delta_k \pm \delta_j;$$

$$\int_{k=1}^{M} \int_{j=1}^{M} Z_k (\delta_k z) \frac{dZ_j (\delta_j z)}{dz} dz = \sum_{\pm} \frac{\delta_j}{2p_{kj}} \left[\left(1 \mp \frac{\delta_k \delta_j}{h_1^2} \right) (1 - \cos p_{kj} L) - \frac{p_{kj}}{h_1} \sin p_{kj} L \right].$$

The application of the integral transformation of the variable to equation (1) gives:

$$\frac{\partial \overline{T}}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \overline{T}}{\partial r} \right) - \delta_k^2 \overline{T}_k(r, \delta_k, t) + \frac{a_k}{\lambda} z_k(\delta_k) w + Q z_k;$$
(5)

$$Qz_k = \left[h_1\left(\sin\delta_k L - \frac{\delta_k}{h_1}\cos\delta_k L\right) + (1 - h_1)\frac{\delta_k}{h_1}\right]T_{0k}.$$
 (6)

Boundary conditions (3) take the form

$$\overline{T}|_{r=0} = 0; \frac{\partial T_k}{\partial r}\Big|_{r=r_3} = -z\mathbf{1}_k \frac{q_k}{\lambda_k} = \overline{q}_k;$$
(7)

$$\left[\lambda_1(T_k)\frac{\partial \overline{T}_k}{\partial r} + \alpha_1(T_k)_1\overline{T}_k\right]_{r=r_4} = z\mathbf{1}_k L_G + \overline{L_G}, \quad z\mathbf{1}_k = \int_0^L Z_k(\delta_k z)dz; \quad (8)$$

$$\overline{T}_k(r,t,\delta_k) = \int_0^L Z(\delta_k z) T(r,t,z) dz, \ \overline{L_G} = \int_0^L Z_k(\delta_k z) N_G(T_k) dz.$$
(9)

Regarding the variable r, we have the Bessel equation within $r \in [r_3, r_4]$:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}_{k}(r,\delta_{k},t)}{\partial r}\right) + \delta_{k}^{2}\overline{T}_{k}(r,\delta_{k},t) = 0$$
(10)

with homogeneous boundary conditions

$$\frac{d\overline{T}_k(r,\delta_k,t)}{dr}\Big|_{r=r_3} = 0; \left[\lambda_1(T_k)\frac{d\overline{T}_k(r,\delta_k,t)}{dr} + \alpha_1(T_k)\overline{T}_k(r,\delta_k,t)\right]\Big|_{r=r_4} = 0.$$

Own functions for the variable take the form:

$$R_{n}(\beta r) = \frac{1}{\left\|R_{n}(\beta_{n}r)\right\|^{2}} [I_{0}(\beta r) - D_{n}(\beta r_{3})J_{0}(\beta r)].$$
(11)

Substitution in (5) for $r = r_4$ in the second boundary condition (9) taking into account the expression for leads to a characteristic equation for finding eigenvalues β_n :

$$h_1[J_1(\beta r_3)I_0(\beta r_4) - I_1(\beta r_3)J_0(\beta r_4)] - \beta[J_1(\beta r_3)I_1(\beta r_4) - I_1(\beta r_3)J_1(\beta r_4)] = 0$$

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Denote by $\gamma = \beta r_4$, $c = r_3/r_4$ and

$$B_m^l(\gamma) = J_m(\gamma c)I_l(\gamma) - I_m(\gamma c)J_l(\gamma); \ l, m = 1, 2.$$

Then to calculate the eigenvalues β_n we will have the following functions

$$q(\gamma) = \frac{\gamma}{r_4} B_1^1(\gamma) - h_1 B_1^0(\gamma) = 0;$$

$$w(\gamma) = \frac{c}{b} \gamma B_0^1(\gamma) - \left(\frac{1}{r_4} + h_1\right) B_1^1(\gamma) - \left(\frac{1}{r_4} \gamma + \frac{h_1}{\gamma}\right) B_1^0(\gamma) + h_1 c B_0^0(\gamma).$$

Coefficients are defined as the norm of eigenfunctions $R_n(\beta_n r)$, $D_n = \frac{I_1(\beta_n r_3)}{J_1(\beta_n r_3)}$:

$$B_n = ||R(\beta_n r)||^2 = \int_{r_3}^{r_4} r R_n^2(\beta_n r) dr = \int_{r_3}^{r_4} [I_0(\beta_n r) - D_n J_0(\beta_n r)]^2 r dr.$$
(14)

Integral transformation of the variable to equation (1) and boundary conditions (3), (4):

$$\frac{\partial \overline{\overline{T}}}{\partial t} + \eta_{nk} \overline{\overline{T}} = \overline{\overline{L}}_G - R_n(r_4) \overline{N}_G; \qquad (15)$$
$$\overline{\overline{L}}_G = \overline{Z_k(0)T_0} + (R_n(r_4)\overline{L}_G + R_n(r_3)Z_R); \ Z_R = z_k q_k / \lambda_1.$$

In the space of Laplace images we have

$$\overline{\overline{T}}_{k}(\eta_{nk},p) = \frac{\overline{L}_{G}}{p(p+\eta_{nk})} - R_{n}(\beta_{n}r_{4})\frac{1}{p+\eta_{nk}}\mathcal{L}\{\overline{N}_{G}[T_{k}^{(0)}(\beta_{n},\delta_{k},t)]\}.$$
 (16)

Since the right-hand side of this expression contains a nonlinear function from T_k (Stefan–Boltzmann condition), $N_G = [T_k(r_4, z, t)/100]^4$ we will look for the solution of equation (16) by an iterative scheme. In the first iteration we obtain the solution of the equation without taking into account the nonlinear function N_G .

$$T_k^{(0)}(r,z,t) = \sum_{n=1}^M \sum_{k=1}^M R_n(\beta_n r) Z_k(\delta_k z) C_{nk}(1 - e^{-\eta_{nk}t}),$$
(17)

where $C_{nk} = \frac{\overline{\overline{L}}_G}{\eta_{nk}}$, $\eta_{nk} = \frac{\lambda_k}{c_v \rho} (\beta_n^2 + \delta_k^2)$.

Substitute the solution (17) taking into account the integral transformations of the variable

$$\overline{N}_{G}[T_{k}^{(0)}(\beta_{n},\delta_{k},t)] = \prod_{m=1}^{4} \sum_{m=1}^{M} \sum_{m=1}^{N} D_{n_{m},k_{m}} \left(1 - e^{-\eta_{n_{m},k_{m}}t}\right);$$
(18)
$$D_{n_{j},k_{j}}rn_{n_{j}}zk_{k_{j}}C_{n_{j},k_{j}}; \ m = [n,k;n_{1},k_{1};n_{2},k_{2};n_{3},k_{3};n_{4},k_{4}]$$

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$$rn_{n_{j}} = R_{n}(\beta_{n}r_{4})R_{n_{1}}(\beta_{n_{1}}r_{4})R_{n_{2}}(\beta_{n_{2}}r_{4})R_{n_{3}}(\beta_{n_{3}}r_{4})R_{n_{4}}(\beta_{n_{4}}r_{4});$$

$$zk_{k_{j}} = \frac{1}{||Z_{k}(\delta_{k}z)||} \int_{k=1}^{N} Z_{k}(\beta_{k}z)Z_{k_{1}}(\beta_{k_{1}}z)Z_{k_{2}}(\beta_{k_{2}}z)Z_{k_{3}}(\beta_{k_{3}}z)Z_{k_{4}}(\beta_{k_{4}}z)dz.$$

Since in this expression the temperature $T_k^{(0)}(r,z,t)$ is in the 4th degree, we will not write this product (the calculation of the corresponding transformations is implemented using the appropriate program in the language C [6]). Write the product for one component, for example, for,. $n_1 = n_2 = n_3 = n_4 = 1$, $k_1 = k_2 = k_3 = k_4 = 1$. Let's mark it as $\overline{N}_{1,1,G}$. Then

$$\overline{N}_{1,1,G} = \frac{1}{100^4} [D_{11}^4 (1 - e^{-\eta_1 t})]^4 = \sum_{l=0}^4 d_l e^{-\sigma_l t},$$
(19)

where $d_l = (-1)^l (D_{1,1}/100)_l^4$; $\sigma_1 = \eta_{1,1}, \sigma_2 = 2\eta_{1,1}, \sigma_3 = 3\eta_{1,1}, \sigma_4 = 4\eta_{1,1}.$

The other components of expression will have a similar form with the change of indices to. We emphasize that the dependence of the nonlinear component of the equation is reflected in the coefficient and remains unchanged in subsequent transformations.

We now apply to (19) an algorithm of equivalent simplification [7], which converts the expressions of the form (19) into a second-order chain in Laplace image space. Then we will obtain

$$\overline{\overline{T}}_{k}(\eta_{nk},p) = \frac{a_{2}^{n,k}}{p} + \frac{a_{0}^{n,k} + a_{1}^{n,k}}{a_{3}^{n,k} + a_{4}^{n,k}p + a_{5}^{n,k}p^{2}}.$$
(20)

In the space of the originals, the expression for temperature will look like $T_{i}(r, z, t) =$

$$= \sum_{n=1}^{M} \sum_{k=1}^{N} R_{n}(\beta_{k}r) Z_{k}(\delta_{k}z) \left[a_{2}^{n,k} + e^{-\alpha^{n,k}t} [\overline{b}_{0}^{n,k} f_{1}(\omega^{n,k}t) + \overline{b}_{1}^{n,k} f_{2}(\omega^{n,k}t)] \right].$$
(21)
Here

Here

$$f_1(\omega^{n,k}t) = \begin{cases} \sin(\omega^{n,k}t) & \omega^{n,k} > 0, \\ \operatorname{sh}(\omega^{n,k}t) & \omega^{n,k} < 0; \end{cases} \quad f_2(\omega^{n,k}t) = \begin{cases} \cos(\omega^{n,k}t) & \omega^{n,k} > 0, \\ \operatorname{ch}(\omega^{n,k}t) & \omega^{n,k} < 0. \end{cases}$$

In the next stages of the iterative procedure we will have the product (for example, for n_j , $j = \overline{1,4}$, k_l , $l = \overline{1,4}$) of functions

$$\overline{N}_{G} = \prod_{l=1}^{4} \overline{\overline{T}}_{k}^{n_{j},k_{l}}(\eta_{n_{j},k_{l}},t) \rightleftharpoons \frac{e_{2}^{n_{j},k_{l}}}{p} + \frac{e_{0}^{n_{j},k_{l}} + e_{1}^{n_{j},k_{l}} p}{e_{3}^{n_{j},k_{l}} + e_{4}^{n_{j},k_{l}} p + e_{5}^{n_{j},k_{l}} p^{2}}$$

The summation in (21) for n_j with the corresponding transformations gives the expression of the body temperature taking into account the nonlinear component of the solution.

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The application of the iterative procedure for calculating the expressions of the form (20) leads to a change in the values of the coefficients $a_j^{n,k}$, $j = \overline{0,5}$, but the structure of this expression remains unchanged.

After the implementation of the iterative procedure for achieving the requirements of accuracy (number of iterations) we obtain the solution of the problem:

$$T_{k}(r,z,t) =$$

$$= \sum_{n=1}^{M} R_{n}(\beta_{n}r) \sum_{k=1}^{N} Z_{k}(\beta_{k}z) \left[\overline{b}_{2}^{n,k} + e^{-\alpha^{n,k}t} (\overline{b}_{0}^{n,k}f_{1}(\omega^{n,k}t) + \overline{b}_{1}^{n,k}f_{2}(\omega^{n,k}t)) \right].$$
(22)

Thus, due to the use of algorithms of equivalent simplification of nonlinear expressions in equations, we obtain the solution of a nonlinear boundary value problem in the class of linear functions.

The main purpose of mathematical modeling of heat transfer in the extruder body is to determine the temperature field on the inner surface of the body on the border with the auger, which determines the heating conditions of the polymer mixture in the loading zone. Based on this, expression (22) takes the form

$$T_{k}(r_{3},z,t) =$$

$$= \sum_{n=1}^{M} R_{n}(\beta_{n}r_{3}) \sum_{k=1}^{N} Z_{k}(\beta_{k}z) \left[\overline{b}_{2}^{n,k} + e^{-\alpha^{n,k}t} [\overline{b}_{0}^{n,k}f_{1}(\omega^{n,k}t) + \overline{b}_{1}^{n,k}f_{2}(\omega^{n,k}t)] \right].$$
(23)

Summation in (23) by eigenfunctions in Laplace image space) on the inner surface of the cylinder gives

$$\widetilde{T}_{k}(r_{3},z,p) = \sum_{k=1}^{N} Z_{k}(\beta_{k}z) \left[\frac{c_{2}^{k}}{p} + \frac{c_{0}^{k} + c_{1}^{k}p}{c_{3}^{k} + c_{4}^{k}p + c_{5}^{k}p^{2}} \right].$$
(24)

The original of this expression is

$$\widetilde{T}_{k}(r_{3},z,t) = \sum_{k=1}^{N} Z_{k}(\beta_{k}z) \bigg[\overline{c}_{2}^{k} + e^{-\xi^{k}t} [\overline{c}_{0}^{k}f_{1}(\chi^{k}t) + \overline{c}_{1}^{k}f_{2}(\chi^{k}t)] \bigg], \qquad (25)$$

Expression (25) serves as a boundary condition on the outer surface of the auger in the boundary value problem of heat transfer in the polymer mixture in the zone of its loading.

SIMULATION RESULTS

In fig. 2, a, b shows the results of modeling the temperature field of the auger body.

In fig. 3, *a*, *b* show the temperature fields on the inner surface of the extruder body in the linear case (first iteration) and taking into account the radiation (third iteration).

Finally, we need to determine the temperature field at the boundary between the inside of the housing and the loaded polymer mixture, i.e. $T_k(r_3, z, t)$ In fig. 3, a, b the graphs of temperature distribution on length of a loading zone is resulted.

With the help of the proposed algorithm for solving the nonlinear boundary value problem, it is possible to develop an algorithm for controlling the temperature field of the housing in order to optimize the process of heating the mixture in the loading zone and melting and crystallization zones of the polymer melt.



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CONCLUSIONS

A mathematical model of induction heating of the extruder body in the manufacture of polymer products, in particular, insulating coating of cables for ultrahigh voltages, which takes into account the convective and radiant heat exchange in the gap inductor-body.

Mathematical modeling of the temperature field of the extruder body during induction heating of the outer surface of the body is performed. The obtained expression for the temperature field on the inner surface of the extruder housing is used as a boundary condition on the outer surface of the auger in the study of heat transfer in the loading zone of the polymer mixture.

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INFORMATION ON THE ARTICLE

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МОДЕЛЮВАННЯ ТЕМПЕРАТУРНОГО ПОЛЯ КОРПУСУ ЕКСТРУДЕРА / О.М. Трофимчук, К.Х. Зеленський, Є.А. Настенко

Анотація. Розглянуто процес індукційного нагрівання корпусу екструдера, температура якого визначає ступінь нагрівання полімерної суміші в зоні заван-

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таження сухої суміші. Сформульовано математичну модель цього процесу з урахуванням променистого теплообміну в зазорі між індуктором і корпусом. Запропоновано ітераційний числово-аналітичний метод розв'язання відповідної крайової задачі про нагрівання корпусу, на першій ітерації якого розв'язується лінійна крайова задача (без урахування променистого теплообміну). На наступних ітераціях розв'язується нелінійна крайова задача. Ітераційний метод грунтується на застосуванні інтегральних перетворень лінійної частини задачі з наступною ітераційною схемою пошуку нелінійної задачі. В основу цієї схеми покладено алгоритми еквівалентного спрощення виразів, отриманих під час розв'язання задачі. Наведено результати математичного моделювання відповідних алгоритмів.

Ключові слова: еквівалентне спрощення, екструдер, індукційне нагрівання, інтегральні перетворення, полімер, функції Бесселя.

МОДЕЛИРОВАНИЕ ТЕМПЕРАТУРНОГО ПОЛЯ КОРПУСА ЭКСТРУДЕРА /

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Аннотация. Рассмотрен процесс индукционного нагрева корпуса экструдера, температура которого определяет степень нагрева полимерной смеси в зоне загрузки сухой смеси. Сформулирована математическая модель этого процесса с учетом лучистого теплообмена в зазоре между индуктором и корпусом. Предложен итерационный численно-аналитический метод решения соответствующей нелинейной краевой задачи о нагреве корпуса, на первой итерации которого решается линейная краевая задача (без учета лучистого теплообмена). На последующих этапах решается нелинейная краевая задача. Итерационный метод основывается на применении интегральных преобразованиях линейной части задачи с последующей итерационной схемой отыскания нелинейной задачи. В основу этой схемы положены алгоритмы эквивалентного упрощения выражений, полученных при решении задачи. Приведены результаты математического моделирования соответствующих алгоритмов.

Ключевые слова: эквивалентное упрощение, экструдер, индукционный нагрев, интегральное преобразование, полимер, функции Бесселя.