МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ, ПРОБЛЕМИ І ТЕХНОЛОГІЇ ДОСЛІДЖЕННЯ СКЛАДНИХ СИСТЕМ

UDC 539.3
DOI: 10.20535/SRIT.2308-8893.2022.2.09

MODELLING NEGATIVE THERMOMECHANICAL EFFECTS IN REINFORCED ROAD STRUCTURES WITH THERMOELASTIC INCOMPATIBILITY OF COATING AND REINFORCEMENT MATERIALS

V.I. GULYAYEV, V.V. MOZGOVYI, N.V. SHLYUN, L.V. SHEVCHUK

Abstract. The phenomena of the formation of local defects and cracks in asphalt concrete pavements of roads and bridges are most often observed in climatic zones with large temperature differences during their seasonal and daily changes. To a large extent, this is due to the heterogeneity of the thermomechanical properties of the materials of the coating layers and the base. To prevent these phenomena, reinforcing rods and meshes are introduced into the coating structure. In this work, using the theory of thermoelasticity, it is shown by the method of mathematical modelling that in cases of incompatibility of the thermomechanical characteristics of asphalt concrete materials and reinforcement, additional localized thermal stresses arise in its small vicinity, which, even at moderate temperatures, can reach critical values and lead to local defects and cracks. Since these defects are latent, they cannot always be detected in practice. The presented results of analytic calculation validated these conclusions. They can be used in both road building and composite design.

Keywords: reinforced asphalt concretes, thermomechanical incompatibility, mathematical modelling, destruction prevention.

INTRODUCTION

The strength and durability of the roadway is largely determined by the intensity of traffic loads and the impact of climatic conditions. Noticeable destruction of road surfaces, bridges, tunnels and dams, as well as other infrastructure facilities in climatic zones with large temperature differences, as a rule, occurs during off-season periods, accompanied by high temperature gradients.

Among the most common types of thermal destruction of the roadway is the appearance of transverse cracks in it, caused by the limiting values of longitudinal stresses at low negative temperatures in the conditions of the impossibility of free shortening of the upper layers. To avoid this effect, so-called “unloading expansion joints” and reinforcement (longitudinal, mesh, etc.) are introduced into the road structure. Such a general strengthening of the roadway with reinforcement leads to an increase in its overall strength, a reduction of deformability, an enlargement of durability, and a decrease in the cost of repair work.

In the theoretical analysis of the effect of reinforcement on the structural strength and the study of the general thermomechanical properties of reinforced (composite) materials and road coatings, the reduced (effective) values of the parameters of combined systems containing inclusions in the form of particles, fibers or rods are mainly determined [4, 6]. In these cases, mainly, models of homogeneous and inhomogeneous spherical particles, including those coated with shell layers, are considered [4]. The cases of ordered [2] and stochastic [10] placement of grains of these particles are singled out, and the reduced values of Young's modulus, Poisson's ratio, thermal conductivity coefficient, and thermal expansion coefficient of the entire system are calculated for them.

Very complex processes of thermal deformation and thermal destruction are observed in the structures of asphalt concrete pavements of roads and bridges [3, 9, 13, 16, 17, 19, 20]. The issues of determining the reduced thermomechanical characteristics of asphalt concrete materials reinforced with particles, fibers and rods are considered in publications [5, 7, 12, 14]. Here, however, these tasks become more complicated, since it is possible to create materials with directional (anisotropic) properties.

In addition, it should be noted that the insertion of reinforcing inclusions from another material into one material can not only improve the generalized characteristics of the entire composite, but under thermal effects it can also be accompanied by the generation of noticeable additional local internal thermal stresses if the thermomechanical characteristics of the composite components are incompatible. For plastic materials, these stresses can lead to local plastic deformations and defects; for brittle materials, to local cracking. Since these defects are localized and latent, they are not always detectable. Therefore, the problem of their theoretical forecasting seems to be relevant.

To simulate these effects, in this work, on the basis of the theory of thermoelasticity, the problem is posed of a planar thermally deformed state of an elastic medium containing an elastic rod of a circular cross section with different thermomechanical parameters. For the case of a change in the temperature of the system by a constant value, an analytical solution of the constitutive equations is constructed, expressions for thermal deformations and thermal stresses are obtained. The general regularities of possible negative influence of the thermomechanical incompatibility of the system parameters on the internal fields of the additional stresses are found. It has been established that the maximum thermal stresses in the medium are realized on the surface of its contact with an elastic inclusion, and they decrease along the radial coordinate in proportion to the square of the distance to the rod axis. The conditions for thermomechanical compatibility of the properties of the medium and the rod are formulated, under which there are no additional thermal stresses in the system. It is shown that in a system with incompatible parameters, additional thermal stresses can be decreased by reducing the radial rigidity of the inclusion through insertion a cylindrical cavity into it.

STATEMENT OF THE PROBLEM

Let us formulate the problem of stationary thermal deformation of an infinite elastic medium 2 (matrix), which is reinforced with rod 1 of circular cross section of radius r_1. Fig. 1 shows a fragment of this system.

Let's use a cylindrical coordinate system $Or\varphi z$, axis Oz of which coincides with the axis of the rod. Let the thermomechanical characteristics of rod 1 and medium 2 be determined, respectively, by the Lame parameters λ_1, μ_1 and λ_2, μ_2 and coefficients of thermal linear expansion α_1 and α_2. The temperature of the system changes steadily by the value ΔT . Let us single out the case when the thermoelastic relative strains $\varepsilon_z^{(i)}(r,\varphi,z)$ of the bodies $i=1,2$ along the Oz axis are equal to zero and the system is in a plane axisymmetric thermally deformed state, described by the equilibrium equations [1, 8, 11, 15, 18]

$$\frac{d\sigma_r^{(i)}}{dr}+\frac{\sigma_r^{(i)}-\sigma_\varphi^{(i)}}{r}=0, \quad (i=1,2), \tag{1}$$

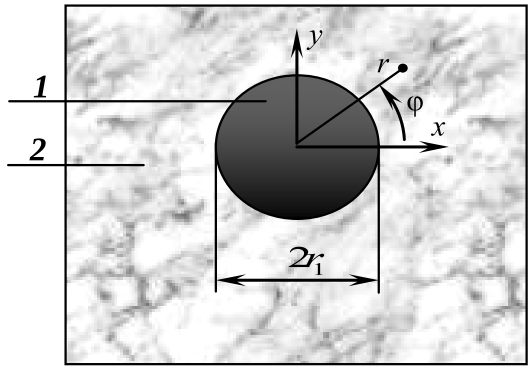

Fig. 1. Planar fragment of an elastic medium with a rod inclusion

where $\sigma_r^{(i)}$, $\sigma_\varphi^{(i)}$ are the normal radial and circumferential stresses of bodies 1 and 2 on the respective areas $r=\text{const}$ and $\varphi=\text{const}$. Let us express normal thermal stresses in terms of strains $\varepsilon_r^{(i)}$, $\varepsilon_\varphi^{(i)}$, $\varepsilon_z^{(i)}$:

$$\sigma_r^{(i)}(r)=(\lambda_i+2\mu_i)\varepsilon_r^{(i)}+\lambda_i(\varepsilon_\phi^{(i)}+\varepsilon_z^{(i)})-(3\lambda_i+2\mu_i)\lambda_i\Delta T;$$

$$\sigma_\phi^{(i)}(r)=(\lambda_i+2\mu_i)\varepsilon_\phi^{(i)}+\lambda_i(\varepsilon_r^{(i)}+\varepsilon_z^{(i)})-(3\lambda_i+2\mu_i)\alpha_i\Delta T;$$

$$\sigma_z^{(i)}(r)=(\lambda_i+2\mu_i)\varepsilon_z^{(i)}+\lambda_i(\varepsilon_r^{(i)}+\varepsilon_\phi^{(i)})-(3\lambda_i+2\mu_i)\alpha_i\Delta T, \quad (i=1,2). \tag{2}$$

Next, we take into account that $\varepsilon_z^{(i)}(r,\varphi,z)=0$. Then expressions (2) will be simplified

$$\sigma_r^{(i)}(r)=(\lambda_i+2\mu_i)\varepsilon_r^{(i)}+\lambda_i\varepsilon_\varphi^{(i)}-(3\lambda_i+2\mu_i)\alpha_i\Delta T;$$

$$\sigma_\phi^{(i)}(r)=(\lambda_i+2\mu_i)\varepsilon_\phi^{(i)}+\lambda_i\varepsilon_r^{(i)}-(3\lambda_i+2\mu_i)\alpha_i\Delta T; \tag{3}$$

$$\sigma_z^{(i)}(r)=\lambda_i(\varepsilon_r^{(i)}+\varepsilon_\phi^{(i)})-(3\lambda_i+2\mu_i)\alpha_i\Delta T, \quad (i=1,2).$$

The deformations used in (3) depend on the radial displacement $u(r)$:

$$\varepsilon_r^{(i)}(r)=\frac{\partial u^{(i)}}{\partial r}, \quad \varepsilon_\varphi^{(i)}(r)=\frac{u^{(i)}}{r}, \quad (i=1,2). \tag{4}$$

Taking into account (3), (4), equation (1) is reduced to the form

$$\frac{d^2u^{(i)}}{dr^2}+\frac{1}{r}\frac{du^{(i)}}{dr}-\frac{1}{r^2}u^{(i)}=0 \qquad (5)$$

for each body $i=1,2$.

Let us represent equation (5) in a more compact form:

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru^{(i)})\right]=0, \quad (i=1,2). \qquad (6)$$

Integrating the left side of equation (6) twice over r , get his solutions

$$u^{(1)}(r)=rC_1+\frac{1}{r}C_2 \text{ at } i=1;$$

$$u^{(2)}(r)=rC_3+\frac{1}{r}C_4 \text{ at } i=2. \qquad (7)$$

The unknown constants C_i $(i=\overline{1,4})$ included here are determined from the boundary conditions and the contact equation for $r=r_1$:

$$u^{(0)}(0)=0\,; \qquad (8)$$

$$u_r^{(1)}(r_1)=u_r^{(2)}(r_1)\,; \qquad (9)$$

$$\sigma_r^{(1)}(r_1)=\sigma_r^{(2)}(r_1)\,; \qquad (10)$$

$$\sigma_r^{(2)}(r)\to 0 \text{ at } r\to\infty\,. \qquad (11)$$

Condition (8) implies

$$C_2=0\,.$$

Using equalities (7), we express the strains and stresses of bodies 1 and 2 in terms of C_i $(i=1,3,4)$:

$$\varepsilon_r^{(1)}(r)=C_1, \quad \varepsilon_\varphi^{(1)}(r)=C_1; \;\; \varepsilon_r^{(2)}(r)=C_3-\frac{1}{r^2}C_4, \quad \varepsilon_\varphi^{(2)}(r)=C_3+\frac{1}{r^2}C_4\,;$$

$$\sigma_r^{(1)}(r)=2(\lambda_1+\mu_1)C_1-(3\lambda_1+2\mu_1)\alpha_1\Delta T; \qquad (12)$$

$$\sigma_r^{(2)}(r)=2(\lambda_2+\mu_2)C_3-\frac{2\mu_2}{r_1^2}C_4-(3\lambda_2+2\mu_2)\alpha_2\Delta T.$$

Condition (11) and the last equality of system (12) imply:

$$C_3=\frac{3\lambda_2+2\mu_2}{2(\lambda_2+\mu_2)}\alpha_2\Delta T\,.$$

Constants C_1 and C_4 are found from the system of equations (9), (10) transformed taking into account equalities (12),

$$C_1=\frac{[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1+(3\lambda_2+2\mu_2)\mu_2\alpha_2]\Delta T}{2(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)},$$

$$C_4=\frac{r_1^2[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{2(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)}.$$

Knowing constants $C_i \; (i=\overline{1,4})$, find the displacement functions:

$$u^{(1)}(r)=\frac{r[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1+(3\lambda_2+2\mu_2)\mu_2\alpha_2]\Delta T}{2(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)}, \quad (0\le r\le r_1),$$

$$u^{(2)}(r)=r\frac{(3\lambda_1+2\mu_2)}{2(\lambda_2+\mu_2)}\alpha_2\Delta T+$$

$$+\frac{r_1^2[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{2(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)}, \quad (r\ge r_1).$$

Note that in the equation for $u^{(2)}(r)$ the first term is the radial displacement in a homogeneous medium 2 in the absence of rod 1, the second term is due to the influence of body 1. It decreases in proportion to radius r .

We also give expressions for the stresses in rod 1:

$$\sigma_r^{(1)}(r)=\sigma_\phi^{(1)}=-\frac{\mu_2[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1`)\alpha_2]\Delta T}{(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)};$$

$$\sigma_z^{(1)}(r)=$$

$$=\frac{[-(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)(\mu_1+\mu_2)\alpha_1+(3\lambda_2+\mu_2)\lambda_1\mu_2\alpha_2]\Delta T}{(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)}, \quad (0\le r\le r_1) \quad (13)$$

and in medium 2:

$$\sigma_r^{(2)}(r)=-\frac{r_1^2}{r^2}\frac{\mu_2[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)};$$

$$\sigma_\phi^{(2)}(r)=\frac{r_1^2}{r^2}\frac{\mu_2[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{(\lambda_1+\mu_1+\mu_2)(\lambda_2+\mu_2)};$$

$$\sigma_z^{(2)}(r)=-\frac{\mu_2(3\lambda_2+2\mu_2)}{(\lambda_2+\mu_2)}\alpha_2\Delta T, \quad (r\ge r_1). \quad (14)$$

Graphs of these functions are shown in Fig. 2. They indicate that additional

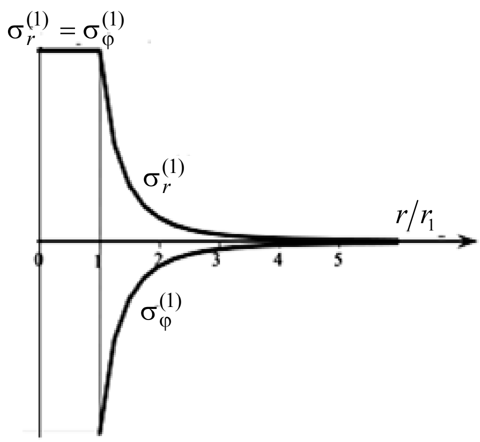

Fig. 2. Graphs of the thermal stresses distribution in the plane of the axial section of the reinforced system under planar thermal deformation

thermal stresses $\sigma_r^{(2)}(r)$, $\sigma_\phi^{(2)}(r)$, caused by the inclusion of reinforcing rod 1 into medium 2, are local in nature and decrease in proportion to the square of the radial coordinate. In addition, they are equal to each other in absolute value and differ in signs, which depend on the ratio of quantities α_1 and α_2. So if $\alpha_1 > \alpha_2$, then, as follows from the form of the numerators of formula (14) with $\Delta T > 0$, $\alpha_1 > \alpha_2$, there are inequalities $\sigma_r^{(2)}(r) < 0$, $\sigma_\phi^{(2)}(r) > 0$, and if $\alpha_1 < \alpha_2$, then vice versa, $\sigma_r^{(2)}(r) > 0$, $\sigma_\phi^{(2)}(r) < 0$. This means that, since asphalt concrete has a lower tensile strength than compressive strength, under any temperature ΔT changes, unfavorable thermal stresses will be realized for $\sigma_r^{(2)}(r)$ or $\sigma_\phi^{(2)}(r)$.

It is also obvious that the values of thermomechanical parameters, at which the numerators of fractions (14) of functions $\sigma_r^{(2)}(r), \sigma_\phi^{(2)}(r)$ are zero, are thermally compatible. Therefore, equality

$$(3\lambda_1 + 2\mu_1)(\lambda_2 + \mu_2)\alpha_1 - (3\lambda_2 + 2\mu_2)(\lambda_1 + \mu_1)\alpha_2 = 0 \qquad (15)$$

represents a condition for the compatibility of the thermomechanical parameters of the matrix and the reinforcing rod.

Condition (15) can be simplified if to replace the Lame parameters λ and μ with modulus of elasticity E and Poisson's ratio υ, using formulas

$$\lambda = \frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)}; \quad \mu = \frac{E}{2(1+\upsilon)}.$$

Then, instead of (15) we have a simpler record of this condition

$$\frac{\alpha_1}{1+\upsilon_2} = \frac{\alpha_2}{1+\upsilon_1}.$$

As an example, consider the case when a fiberglass reinforcing rod of radius r_1 with thermomechanical parameters $\lambda_1 = 30.2$ GPa, $\mu_1 = 12.9$ GPa, $\alpha_1 = 21 \cdot 10^{-6}\ \text{K}^{-1}$ is located in an asphalt concrete medium with parameters $\lambda_2 = 1.39$ GPa, $\mu_2 = 2.08$ GPa, $\alpha_2 = 10 \cdot 10^{-6}\ \text{K}^{-1}$. It is accepted that $\Delta T = -20^\circ \text{K}$. With these data, the thermal stresses in the system amounted to $\sigma_r^{(1)} = \sigma_\phi^{(1)} = 1.298$ MPa, $\sigma_z^{(1)} = 15.541$ MPa, $\sigma_r^{(2)} = 1.298 \frac{r_1^2}{r^2}$ MPa, $\sigma_z^{(2)} = -1.298 \frac{r_1^2}{r^2}$ MPa, $\sigma_z^{(2)} = 0.9986$ MPa.

If we take into account that the ultimate strength of asphalt concrete in compression is $5 \div 20$ MPa, and in tension it turns out to be several times less than these values, then we can conclude that under the considered conditions, additional thermal stresses in asphalt concrete, caused by the insertion of a fiberglass reinforcing rod into it, can lead to the occurrence of local defects in its small neighbourhood.

REDUCING THE LEVEL OF ADDITIONAL THERMAL STRESSES ON TUBULAR REINFORCING RODS

As can be seen from equalities (13), (14), additional stresses $\sigma_r^{(i)}$, $\sigma_\phi^{(i)}$ $(i=1,2)$ are determined not only by the values of coefficients α_1, α_2, but also by elasticity parameters λ_i, μ_i $(i=1,2)$, which are included in the numerators of fractions (13), (14) in the third powers, and in the denominators — in the second ones. Therefore, additional thermal stresses in the system increase with increasing λ_i, μ_i $(i=1,2)$ or, for example, with an increase in the radial rigidity of rod 1 (while maintaining its axial strength and rigidity). Conversely, they decrease as this stiffness decreases. Given this property, we can propose to use tubular rods as reinforcement in asphalt concrete pavements (Fig. 3). Let us investigate the thermally stressed state in this case. Let r_1 and r_2 be the inner and outer radii of pipe 1, respectively, the dimensions of medium 2 are unlimited. Let us assume, as above, that λ_i, μ_i, α_i $(i=1,2)$ be the thermomechanical parameters of bodies 1 and 2, ΔT — difference in body temperature in the initial and final states. Let us find the functions of thermal stresses in the system for the case of its axisymmetric planar thermally deformed state.

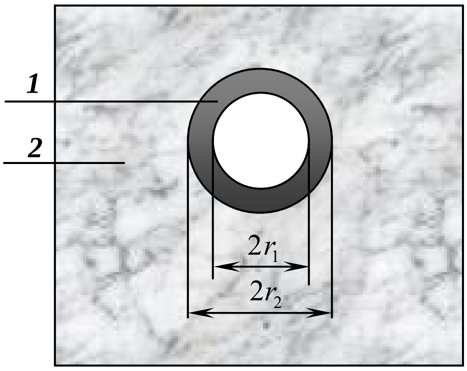

Fig. 3. Planar fragment of an elastic medium with a tubular inclusion

Similarly to the case of a solid rod, the equations of thermoelasticity of the system have form (1) – (6). The solution of these equations is again formulated in the form of functions of radial displacements of body 1

$$u_1(r) = rC_1 + \frac{1}{r}C_2 \quad (r_1 \le r \le r_2),$$

and for medium 2

$$u_2(r) = rC_3 + \frac{1}{r}C_4 \quad (r \ge r_2).$$

At the same time, constants C_i $(i=\overline{1,4})$ are found from the conditions:

$$\sigma_r^{(1)}(r_1) = 0; \;\; u^{(1)}(r_2) = u^{(2)}(r_2);$$

$$\sigma_r^{(1)}(r_2) = \sigma_r^{(2)}(r_2); \;\; \sigma_r^{(2)}(r) \to 0 \;\; \text{at } r \to \infty.$$

After appropriate substitutions, these equations are reduced to the form:

$$2(\lambda_1 + \mu_1)C_1 - \frac{2\mu_1}{r_1^2}C_2 - (3\lambda_1 + 2\mu_1)\alpha_1 \Delta T = 0;$$

$$C_1 + \frac{1}{r_2^2}C_2 - C_3 - \frac{1}{r_2^2}C_4 = 0;$$

$$2(\lambda_1+\mu_1)C_1-\frac{2\mu_1}{r_2^2}C_2-(3\lambda_1+2\mu_1)\alpha_1\Delta T-$$

$$-2(\lambda_2+\mu_2)C_3+\frac{2\mu_2}{r_2^2}C_4+(3\lambda_2+2\mu_2)\alpha_2\Delta T=0;$$

$$2(\lambda_2+\mu_2)C_3-(3\lambda_2+2\mu_2)\alpha_2\Delta T=0. \qquad (16)$$

From the last equation of this system, we obtain

$$C_3=\frac{(3\lambda_2+2\mu_2)}{2(\lambda_2+\mu_2)}\alpha_2\Delta T.$$

Next, from the remaining equations of system (16) we find:

$$C_1=\frac{\mu_1}{r_1^2}\frac{[-(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1+(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{2(\lambda_1+\mu_1)^2(\lambda_2+\mu_2)\left[\frac{\mu_1}{r_1^2(\lambda_1+\mu_1)}+\frac{1}{r_2^2}+\frac{\mu_1}{\mu_2}\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\right]}+$$

$$+\frac{(3\lambda_1+2\mu_1)}{2(\lambda_1+\mu_1)}\alpha_1\Delta T;$$

$$C_2=\frac{[-(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1+(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{2(\lambda_1+\mu_1)(\lambda_2+\mu_2)\left[\frac{\mu_1}{r_1^2(\lambda_1+\mu_1)}+\frac{1}{r_2^2}+\frac{\mu_1}{\mu_2}\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\right]};$$

$$C_4=\frac{r_2^2\mu_1}{\mu_2}\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\frac{[(3\lambda_1+2\mu_1)(\lambda_2+\mu_2)\alpha_1-(3\lambda_2+2\mu_2)(\lambda_1+\mu_1)\alpha_2]\Delta T}{2(\lambda_1+\mu_1)(\lambda_2+\mu_2)\left[\frac{\mu_1}{r_1^2(\lambda_1+\mu_1)}+\frac{1}{r_2^2}+\frac{\mu_1}{\mu_2}\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\right]}.$$

Using the found constants, you can build expressions for displacements

$$u(r)=rC_1+\frac{1}{r}C_2,\ (r_1\le r\le r_2);\quad u(r)=rC_3+\frac{1}{r}C_4,\ (r\ge r_2)$$

and stresses

$$\sigma_r^{(1)}(r)=2(\lambda_1+\mu_1)C_1-\frac{2\mu_1}{r^2}C_2-(3\lambda_1+2\mu_1)\alpha_1\Delta T;$$

$$\sigma_\phi^{(1)}(r)=2(\lambda_1+\mu_1)C_1+\frac{2\mu_1}{r^2}C_2-(3\lambda_1+2\mu_1)\alpha_1\Delta T;$$

$$\sigma_z^{(1)}(r)=2\lambda_1C_1-(3\lambda_1+2\mu_1)\alpha_1\Delta T,\quad (r_1\le r\le r_2)$$

in rod 1 and

$$\sigma_r^{(2)}(r)=-\frac{2\mu_2}{r^2}C_4;\quad \sigma_\phi^{(2)}(r)=-\frac{2\mu_1}{r^2}C_2;$$

$$\sigma_z^{(2)}(r)=2\lambda_2C_3-(3\lambda_2+2\mu_2)\alpha_2\Delta T,\ (r\ge r_1)$$

in medium 2.

Table shows the stress values $\sigma_r^{(1)}$, $\sigma_\phi^{(1)}$, $\sigma_z^{(1)}$ on surfaces $r=r_1$ and $r=r_2$ of fiberglass body 1 and stresses $\sigma_r^{(2)}$, $\sigma_\phi^{(2)}$, $\sigma_z^{(2)}$ on surface $r=r_2$ of asphalt concrete medium 2 for relations $r_1/r_2 = 0.5,\ 0.75,$ and 0.9 at the values of the thermomechanical parameters given above and the temperature difference $\Delta T = -20$ K.

Values of thermal stresses in the medium reinforced with a tubular rod

Types of thermal stresses	r_1/r_2			
	0.0	0.5	0.75	0.9
$\sigma_r^{(1)}(r_1)$ MPa	1.2981	0	0	0
$\sigma_\phi^{(1)}(r_1)$ MPa	1.2981	3.2455	4.5766	7.3764
$\sigma_z^{(1)}(r_1)$ MPa	15.541	15.569	16.235	17.215
$\sigma_r^{(1)}(r_2)$ MPa	1.2981	1.2171	1.0011	0.7008
$\sigma_\phi^{(1)}(r_2)$ MPa	1.2981	2.0285	3.5742	6.6752
$\sigma_z^{(1)}(r_2)$ MPa	15.541	15.569	16.235	17.215
$\sigma_r^{(2)}(r_2)$ MPa	1.2981	1.2171	1.0011	0.7008
$\sigma_\phi^{(2)}(r_2)$ MPa	-1.2981	-1.2171	-1.0011	-0.7008
$\sigma_z^{(2)}(r_2)$ MPa	0.9986	0.9986	0.9986	0.9986

The question of the distribution in the radial direction of additional thermal stresses $\sigma_r^{(i)}(r)$, $\sigma_\phi^{(i)}(r)$ deserves a special interest. Fig. 4 shows the graphs of these functions for case $r_1/r_2 = 0.75$.

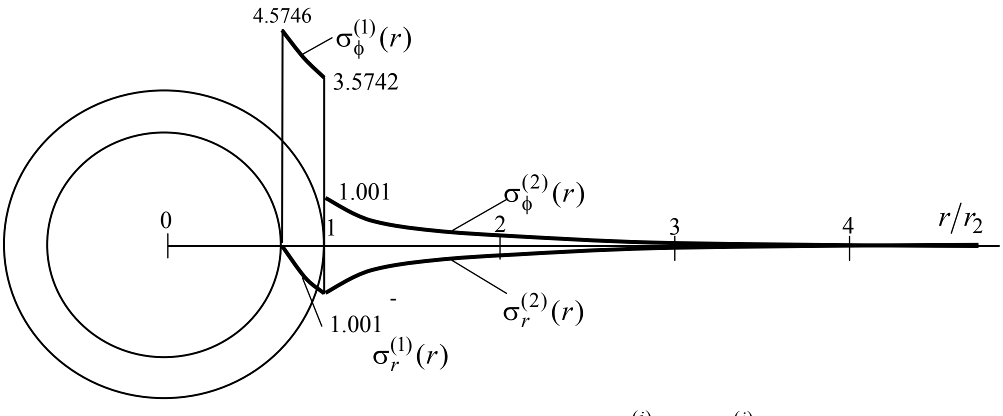

Fig. 4. Graphs of distribution of thermal stresses $\sigma_r^{(i)}(r)$, $\sigma_\phi^{(i)}(r)$ (MPa) for case $r_1/r_2 = 0{,}75$

As can be seen, additional thermal stresses $\sigma_r^{(2)}(r)$, $\sigma_\phi^{(2)}(r)$ in the asphalt concrete have the highest values on the contact surface $r = r_2$ and they decrease rapidly in the radial direction.

An analysis of the above results indicates that the replacement of a solid reinforcing rod with a tubular one leads to a noticeable decrease in additional local thermal stresses in the matrix medium, although the thermal stresses in the rod increase somewhat. This effect becomes more noticeable as the thickness of the tube rod decreases.

CONCLUSION

1. The problem associated with the modelling of the formation of additional thermal stresses, defects and destructions in the medium of an asphalt concrete pavement with the insertion of a reinforcing rod into it is considered. On the basis of thermoelasticity methods, the system of differential equations is formed for a plane axisymmetric deformed state of an infinite cylindrical elastic body in an infinite elastic medium under condition of a change in the temperature of the system.

2. An analytical solution of the formulated equations is constructed in a closed form, which determines the additional thermal displacements, additional strains and stresses in the system. It is shown that additional thermal stresses have the highest values on the contact surface of the reinforcing rod and the coating array and decrease in inverse proportion to the square of the distance from this surface. It has been established that the values of these stresses enlarge with an increase in the thermomechanical incompatibility of the system materials and the radial stiffness of the rod. On the example of asphalt concrete reinforced with a fiberglass rod, it was demonstrated that even with moderate temperature changes, additional thermal stresses in asphalt concrete can reach critical values.

3. A method is proposed for reducing additional thermal stresses by reducing the radial stiffness of the reinforcement by replacing a solid rod with a hollow tube. Theoretical modelling of this effect showed that with a decrease in the tube wall thickness, the decrease in additional contact thermal stresses in asphalt concrete becomes significant.

REFERENCES

1. T.G. Beleicheva and K.K. Ziling, "Thermoelastic axisymmetric problem for a two-layer cylinder", *J. Appl. Mech. Technical Physics*, **19**, pp. 108–113, 1978.
2. Boubaker Fetiza Ali, Khedoudja Soudani, and Smail Haddadi, "Effect of waste plastic and crumb rubber on the thermal oxidative ageing of modified bitumen", *Road Mat. and Pav.*, vol. 23, issue 1, pp. 222–233, Dec. 2022.
3. Brian Hill et al., "Evaluation of low temperature viscoelastic properties and fracture behavior of bio-asphalt mixtures", *International Journal of Pavement Engineering*, vol. 19, issue 4, pp. 362–369, 2018.
4. R.M. Christensen and K.H. Lo, "Solutions for effective shear properties in three-phase sphere and cylinder models", *J. Mech. Phys. Solids*., vol. 27, pp. 315–330, 1979.
5. Christian Karch, "Micromechanical analysis of thermal expansion coefficient", *Modeling and Numerical Simulation of Material Science*, vol. 3, pp. 1–15, 2014.
6. R.M. Christiansen, *Mechanics of Composite Materials*. Wiley: New York, NY, USA, 1979.
7. Gholamali Shafabakhsh, Mohammadreza Aliakbari Bidokhti, and Hassan Divandan, "Evaluation of the performance of SBS/Nano-Al2O3 composite-modified bitumen at high temperature", *Road Mat. and Paves*, vol. 22(11), pp. 2523–2537, 2021.
8. R.B. Hetnarski and J. Ignaczak, *Mathematical Theory of Elasticity*. New York: Taylor and Francis, 2004.
9. Jorge Pais et al., "The adjustment of pavement deflections due to temperature variations", *International Journal of Pavement Engineering*, vol. 21, issue 13, pp. 1585–1594, 2020.
10. J.W. Ju and T.M. Chen, "Effective elastic moduli of two-phase composites containing randomly dispersed spherical inhomogeneities", *Acta Mech.*, 103(1), pp. 123–144, 1994.
11. A.D. Kovalenko, *Thermoelasticity: Basic Theory and Applications*. The Netherlands: Wolters-Noordhoff Groningen, 1972.
12. Marcela Fiedlerova, Petr Jisa, and Kamil Stepanek, "Using various thermal analytical methods for bitumen characterization", *International Journal of Pavement Research and Technology*, vol. 14, issue 4, pp. 459–465, 2021.

13. Md Amanul Hasan and Rafiqul A. Tarefder, "Development of temperature zone map for mechanistic empirical (ME) pavement design", *Journal of Pavement Research and Technology*, vol. 11, issue 1, pp. 99–111, 2018.
14. Mirosław Graczyk, Józef Rafa, and Adam Zofka, "Pavement modelling using mechanical and thermal homogenization of layered systems", *Roads and Bridges - Drogi i Mosty*, 17(2), pp. 141–157, 2018.
15. W. Nowacki, *Thermoelasticity*; 2 nd ed. Oxford: PWN – Polish Scientific Publishers, Warsaw and Pergamon Press, 1986.
16. Dian M. Setiawan, "The role of temperature differential and subgrade quality on stress, curling, and deflection behavior of rigid pavement", *Journal of the Mechanical Behavior of Materials*, vol. 29, issue 5–6, id.10, 12 p.
17. Tarn Minh Phan, Tri Ho Minh Le, and Dae-Wook Park, "Evaluation of cracking resistance of healed warm mix asphalt based on air-void and binder content", *Road Materials and Pavement Design*, vol. 23, issue 1, pp. 47–61, 2022.
18. D.E. Carlson, "Thermoelasticity", in *Truesdell C. (ed) Encyclopedia of physics*, vol. Vla/2: mechanics of solids, Springer, Berlin, 1972.
19. Yang Liu, Zhendong Qian, and Dong Zheng, "Meng Zhang Interlaminar thermal effect analysis of steel bridge deck pavement during gussasphalt mixture paving", *International Journal of Pavement Engineering*, vol. 20, issue 11, pp. 1323–1335, 2019.
20. Yingjun Jiang, Changqing Deng, Zhejiang Chen, and Yuhang Tian, "Evaluation of the cooling effect and anti-rutting performance of thermally resistant and heat-reflective pavement", *International Journal of Pavement Engineering*, vol. 21, issue 4, pp. 447–456, 2020.

Received 02.05.2022

INFORMATION ON THE ARTICLE

Valery I. Gulyayev, ORCID: 0000-0002-5388-006X, National Transport University, Ukraine, e-mail: valery@gulyayev.com.ua

Volodymyr V. Mozgovyy, ORCID: 0000-0002-1032-8048, National Transport University, Ukraine, e-mail: mozgoviy@gmail.com

Nataliia V. Shlyun, ORCID: 0000-0003-1040-8870, National Transport University, Ukraine, e-mail: nataliyashlyun@gmail.com

Lyudmyla V. Shevchuk, ORCID: 0000-0002-5748-9527, National Transport University, Ukraine, e-mail: ludmilashevchuk25@gmail.com

НЕГАТИВНІ ТЕРМОМЕХАНІЧНІ ЕФЕКТИ В АРМОВАНИХ ДОРОЖНІХ КОНСТРУКЦІЯХ ЗА ТЕРМОПРУЖНОЇ НЕСУМІСНОСТІ МАТЕРІАЛІВ ПОКРИТТЯ ТА АРМАТУРИ / В.І. Гуляєв, В.В. Мозговий, Н.В. Шлюнь, Л.В. Шевчук

Анотація. Явища утворення локальних дефектів і тріщин в асфальтобетонних покриттях автомобільних доріг та мостів найчастіше спостерігаються у кліматичних зонах з великими перепадами температур за їх сезонних та добових змін. Значною мірою це зумовлено неоднорідністю термомеханічних властивостей матеріалів шарів покриттів та основи. Для попередження цих явищ в конструкції покриттів уводять армувальні стрижні і сітки. У роботі методами теорії термопружності показано, що у випадках несумісності термомеханічних характеристик матеріалів асфальтобетону та арматури в її малому околі виникають додаткові локалізовані термонапруження, які навіть за помірних значень перепадів температури можуть досягати критичних значень та призводити до локальних дефектів і тріщин. Оскільки ці дефекти мають прихований характер, їх не завжди можна виявляти.

Ключові слова: асфальтобетонне покриття, стрижнева арматура, термомеханічна несумісність, концентрація термонапруг.