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ON SOME METHODS FOR SOLVING THE PROBLEM OF POWER DISTRIBUTION OF DATA TRANSMISSION CHANNELS TAKING INTO ACCOUNT FUZZY CONSTRAINTS ON CONSUMPTION VOLUMES

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Abstract. The article deals with the mathematical formulation of the problem of optimal distribution of the power of data transmission channels in information and computer networks with a three-level architecture and fuzzy restrictions on consumption volumes. An efficient algorithm has been developed for solving the problem, the peculiarity of which is the inability to meet the end user's needs at the expense of the resources of different suppliers. A standard solution method based on a fuzzy optimization problem of mathematical programming is considered. A constructive variant of finding a solution based on the backtracking method is proposed. Computational experiments have been carried out. The developed approach was used to determine the optimal configuration of a three-level information and computer network with a given number of communication servers.

Keywords: data transfer, power distribution, fuzzy constraints, optimal solution, backtracking algorithm.

INTRODUCTION

The tasks of finding optimal solutions arise in the process of development and practical implementation of methods for effective management of various organizational, technological and information systems.

An important characteristic of optimization problems is the desire to find the optimal solution (optimality principle). In practice, there are a number of constraints that do not allow finding such a solution. In these cases, the question is raised of finding not optimal, but rational (compromise, effective) solutions that satisfy the problem statement. It is often necessary to find a compromise between the effectiveness of solutions and the cost of finding them. Serious difficulties arise when solving optimization problems under conditions of incomplete information, as well as in the case when random or subjective factors (parameters) play a significant role.

One of the applied problems in which there can be uncertainty in setting the parameters is the problem of distributing the limited capacities of data transmission channels between different nodes of the Internet providers network. Suppose that there is a local computer network of an enterprise (organization, educational institution) that provides users with access to the Internet. User access to the global network and obtaining the necessary information is carried out using several communication servers located on the territory of the information and computer center of the enterprise and connected by high-speed external communication channels with Internet providers. The bandwidth levels of the servers are within the bandwidth (bandwidth) of the local network (for example, 1Gb per second).

It is assumed that the network implements the conditions for efficient channel switching (relative to their bandwidth), which are provided by programmable network devices (communication servers, routers). The structure of the network and the information distributed in it in the general case can be very diverse. In this case, we consider the problem of distribution of limited capacities with the following constraints:

- information is distributed from the provider to subscribers (nodes) through switching servers via communication channels with a bandwidth that takes into account the specified bandwidth;
- each network subscriber is serviced by one switching server;
- the throughput of receiving information for switching nodes and subscribers is limited both from above (fundamental limitations of the provider's capabilities) and from below (the minimum need for subscribers to receive information).

The problem of determining the bandwidth of an external connection is considered, which makes it possible to maximize the total bandwidth of user communication channels by changing the total power of communication servers, taking into account both the needs and wishes of subscribers (users) and the capabilities of the information and computing center.

The solution of the formulated problem was considered in [1–7] on the basis of solving problems of optimal resource allocation. The problems of efficient use of a homogeneous resource were considered using the example of time distribution in the form of a classical problem of distributing resources of a given volume over a set of categories (works) [8]. The setting of such tasks consists in finding a cost plan for the available resource (such a resource is most often considered time) for the execution of a group of tasks, in which the total (final) use of the resource is optimal.

In a number of papers [9–11] to find a solution, an approach is proposed that uses multi-index problems of the transport type [11]. In the noted works, meaningful formulations of such applied problems are given and their mathematical models are constructed.

When solving applied multi-index optimization problems, special interest is given to formulations related to the class of problems of integer linear programming [11]. One of the approaches to the development of algorithms for solving such optimization problems is the use of streaming methods. Known efficient flow algorithms [12] make it possible to construct solution methods that have acceptable estimates of computational complexity compared to estimates of general methods for solving linear programming problems.

Solutions obtained on the basis of models of three-index transport problems [10] allow solving the problem of distribution of a homogeneous resource for cases where the cost and resource consumption factors are known a priori.

In [13,14], a model of a two-level production and transport problem was considered with a criterion that takes into account the optimal cost indicators for the production and transportation of resources, the volumes of production and consumption of which are given.

PROBLEM FORMALIZATION OF THE DATA TRANSMISSION CHANNELS OPTIMAL DISTRIBUTION POWER

An information and computer network is considered, including N_1 data transmission channels (global network providers), N_2 communication servers and N_3

end users (subscribers). We denote by A_i^+, $i=\overline{1,N_1}$, the values of the maximum bandwidth of the data transmission channel that provider *i*, $i=\overline{1,N_1}$, is able to provide; B_j^+, $j=\overline{1,N_2}$ — the value of the maximum bandwidth of the data transmission channel that the communication node *j*, $j=\overline{1,N_2}$, can provide; C_k^-, C_k^+, $k=\overline{1,N_3}$ — values of the minimum and maximum bandwidth of the data transmission channel, which must be provided to the subscriber *k*, $k=\overline{1,N_3}$; t_k — throughput of the *k*-th subscriber station, $k=\overline{1,N_3}$. Then, assuming that the power distribution of communication channels satisfies the conditions of additivity and proportionality, we can consider the problem of distributing a limited homogeneous resource (bandwidth of communication channels) with transport-type constraints in order to find the optimal data transmission plan. This ensures the effective functioning of the system for providing users with Internet access, which consists in finding the optimal values of data transmission bandwidths T_i of the *i*-th information provider (provider), $i=\overline{1,N_1}$, and the optimal values of the bandwidths t_k of using local communication channels of the *k*-th user, $k=\overline{1,N_3}$.

Formally, the statement of problem can be written as

$$\max t_1;\ \max t_2;\ \ldots \max t_{N_3},$$

with the following constraints

$$\sum_{k=1}^{N_3} t_k \le \sum_{i=1}^{N_1} A_i^+;$$

$$t_k \le B_j^+,\ j=\overline{1,N_2},\ k=\overline{1,N_3};\quad C_k^- \le t_k \le C_k^+,\ k=\overline{1,N_3};$$

$$\sum_{j=1}^{N_2} B_j^+ \le \sum_{i=1}^{N_1} A_i^+ \le \sum_{k=1}^{N_3} C_k^+ .$$

SOLVING METHODS OF THE PROBLEM OF DATA TRANSMISSION CHANNELS OPTIMAL DISTRIBUTION POWER TAKING INTO ACCOUNT FUZZY CONSTRAINTS ON CONSUMPTION VOLUMES

Let's assume that the needs of network subscribers to increase the speed of obtaining one or another amount of information are known. The wishes (preferences) of subscribers are set regarding a possible increase in consumption volumes (bandwidths) for transmitting information from the provider to the user node. To implement the changes, it is necessary to update the capacities of the switching servers of the network by deploying new, more powerful computers or by increasing the number of existing servers. In other words, it is necessary to conduct a study on updating the resources of the server park of the information and computing center, which makes it possible to increase the total bandwidth of a group of switching servers. At the same time, the value of the total capacity of servers, both in the case of an increase in the capacity of the existing fleet of computers, and in the case of an increase in the number of servers, is assumed to be the same.

If the values of consumption parameters are random variables with known distribution functions, then it can be solved by stochastic programming methods. However, in practice these parameters are often unknown and only the range of possible values can be determined for them. A problem of this type can be called a problem with multiple values of the coefficients. Within the framework of this problem, it makes no sense to talk about maximizing the objective function, since the values of this function are not numbers, but sets of numbers. In this case, it is necessary to find out what preference relation this function generates on the set of alternatives, and then determine which products should be considered rational in the sense of this preference relation.

The next step on the way of detailing and refining the model considered here is the description of the problem parameters in the form of fuzzy sets (numbers) [15]. Additional information is introduced into the model in the form of a membership function of these fuzzy sets. These functions can be considered as a way for an expert to approximate his unformalized idea of the real value of a given parameter. Membership function values are the weights that experts assign to the various possible values of this parameter.

Fuzzy sets are a mathematical model of object classes with fuzzy or blurry boundaries. In other words, an element can have some degree of membership in the set, and it is intermediate between full membership and complete non-membership.

Traditional (ordinary) set theory can be viewed as a special case of fuzzy set theory. An ordinary subset A of a set X can be represented as a fuzzy set, which is given by the characteristic function $\chi_A : X \to \{0,1\}$:

$$\chi_A(x) = \begin{cases} 0: & x \notin A; \\ 1: & x \in A. \end{cases}$$

In accordance with the idea of Zadeh [15], a fuzzy subset of a given universal set X is formulated as follows.

Definition 1. A fuzzy subset $\widetilde{A}$ of the universal set X, is a collection of pairs $\widetilde{A} = \{(\mu_{\widetilde{A}}(x), x)\}$, where $\mu_{\widetilde{A}}(x) : X \to [0,1]$ is the mapping of the set X into the unit segment [0,1], which is called the membership function of the fuzzy set.

The value of the membership function $\mu_{\widetilde{A}}(x)$ for an element $x \in X$ is called the degree of membership. The interpretation of the degree of membership $\mu_{\widetilde{A}}(x)$ is a subjective measure of how much an element $x \in X$ corresponds to a concept, the meaning of which is formalized by a fuzzy set $\widetilde{A}$.

Let $X = R^1$ is a universal set.

Definition 2. [16] A fuzzy triangular number (triplet) $\widetilde{A}$ is an ordered triplet of numbers *(a, b, c)*, $a \le b \le c$, defining a membership function $\mu_{\widetilde{A}}(x)$ of the form:

$$\mu_{\widetilde{A}}(x) = \frac{x-a}{b-a}, \ x \in [a,b]; \ \mu_{\widetilde{A}}(x) = \frac{c-x}{c-b}, \ x \in [b,c]; \ x \notin [a,c]. \qquad (1)$$

A fuzzy triangular number of the form (a,b,b), called a left fuzzy triangular number, is determined by the membership function of the form:

$$\mu_{\tilde{A}}(x) = 0,\ x < a\,;\ \mu_{\tilde{A}}(x) = \frac{x-a}{b-a},\ \ x \in [a,b]\,;\ \mu_{\tilde{A}}(x) = 1,\ \ x > b\,,$$

and the fuzzy triangular number of the form *(b, b, c)*, called the right fuzzy triangular number, is the membership function $x \in R^n$:

$$\mu_{\tilde{A}}(x) = 1,\ \ x < b\,;\ \mu_{\tilde{A}}(x) = \frac{c-x}{c-b},\ \ x \in [b,c]\,;\ \mu_{\tilde{A}}(x) = 0,\ x > c\,. \qquad (2)$$

After such clarification, we can proceed to the next statement of the problem of fuzzy mathematical programming [17]. A linear view model is specified

$$\sum_{j=1}^{n} \tilde{c}_j x_j \to \max\,, \qquad (3)$$

in which the values of the coefficients $\tilde{c}_j$, $j = \overline{1,n}$, are given fuzzy in the form of fuzzy sets of given universal sets. In addition, there are constraints:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i\,,\ i = \overline{1,m}\,,\ x_j \ge 0\,,\ j = \overline{1,n}\,, \qquad (4)$$

and the values of the coefficients $\tilde{a}_{ij}$, $\tilde{b}_i$, $i = \overline{1,m}$, $j = \overline{1,n}$, are also described in the form of the corresponding fuzzy sets. It is required to make a rational choice of a solution $x \in R^n$, that, in a certain sense, maximizes the given fuzzy linear form (3).

We call such a statement of the fuzzy optimization problem a linear programming problem with fuzzy parameters. One of its variants is a problem with fuzzy resource constraints on the right side.

Consider now a linear programming problem with a given goal function

$$\max_x \sum_{j=1}^{n} c_j x_j \qquad (5)$$

and fuzzy constraints on resources of the form

$$\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i\,,\ \ i = \overline{1,m}\,,\ \ x \ge 0;\ x \in R^n\,, \qquad (6)$$

where the right parts of constraints (6) are given as fuzzy right triangular numbers with corresponding membership functions of the form (1). Here, the allowable deviations determine the values of the boundary changes of the model resources.

This formulation does not restrict the general form of optimization problems with fuzzy constraints [18, 19]. Indeed, one can consider a linear programming problem with fuzzy resources in the form of an optimization problem for the goal function (5) in the presence of a system of mixed constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \ge \tilde{b}_i\,,\ \ i = \overline{1,m_1}\,;\ \ \sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i\,,\ \ i = \overline{m_1+1,m_2}\,;$$

$$\sum_{j=1}^{n} a_{ij} x_j = \tilde{b}_i\,,\ \ i = \overline{m_2+1,m}\,,$$

where the right parts of the first m_1 constraints are given by left fuzzy triangular numbers $\tilde{b}_i = (b_i - b_i^0, b_i, b_i)$, $b_i^0 \ge 0$, $i = \overline{1,m_1}$, the right parts of the next group of constraints are given by right fuzzy triangular numbers $\widetilde{b}_i = (b_i, b_i, b_i + b_i^0)$, $b_i^0 \geq 0$, $i = \overline{m_1 + 1, m_2}$, and the right parts of the last $m - m_2$ constraints are given by fuzzy triangular numbers $\widetilde{b}_i = (b_i - b_i^l, b_i, b_i + b_i^r)$, with allowable deviations $0 \leq b_i^l \leq b_i$, $b_i^r \geq 0$, $i = \overline{m_2 + 1, m}$.

This LP model can be rewritten in the form (4)–(5) by replacing the first m_1 conditions with the next constraints $\sum_{j=1}^{n}(-a_{ij})x_j \leq -\widetilde{b}_i$, $\widetilde{b}_i = (-b_i, -b_i, -b_i + b_i^0)$, $i = \overline{1, m_1}$, and the last $m - m_2$ conditions — with a system of constraints $\sum_{j=1}^{n}(-a_{ij})x_j \leq -\widetilde{b}_i$, $\widetilde{b}_i = (-b_i, -b_i, -b_i + b_i^l)$; $\sum_{j=1}^{n} a_{ij}x_j \leq \widetilde{b}_i$, $\widetilde{b}_i = (b_i, b_i, b_i + b_i^r)$; $i = \overline{m_2 + 1, m}$. Thus, we can assume that the general form of a linear programming problem with fuzzy resource constraints on the right side is given by model (5)–(6).

The optimization problem under consideration can be solved as a parametric linear programming problem [20]. This method is universal, not always taking into account the specifics of the task.

We use an approach based on the defuzzification of problem (5)–(6). To do this, we calculate the optimal values of the levels of the objective function Z_l and Z_u by solving two linear programming problems:

$$Z_l = \max_x \sum_{j=1}^{n} c_j x_j \tag{7}$$

with constraints

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = \overline{1, m}, \quad x \geq 0; \; x \in R^n; \tag{8}$$

$$Z_u = \max_x \sum_{j=1}^{n} c_j x_j \tag{9}$$

under condition

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i + b_i^0, \quad i = \overline{1, m}, \; x \geq 0; x \in R^n. \tag{10}$$

Let be $L = \min(Z_l, Z_u)$, $U = \max(Z_l, Z_u)$. The fuzzy set of optimal values of problem (5)–(6) specified in R^n (we denote it by $\widetilde{G}$) is described by the membership function of the form:

$$\mu_{\widetilde{G}}(x) = \begin{cases} 0, & \sum_{j=1}^{n} c_j x_j < L; \\ (\sum_{j=1}^{n} c_j x_j - L)/(U - L), & L \leq \sum_{j=1}^{n} c_j x_j \leq U; \\ 1, & \sum_{j=1}^{n} c_j x_j > U, \end{cases} \tag{11}$$

and the fuzzy sets of each constraint (we denote them by $\widetilde{F}_i$, $i=\overline{1,m}$) from (6) are determined by the membership functions:

$$\mu_{\widetilde{F}_i}(x)=\begin{cases}1, & \sum_{j=1}^{n} a_{ij}x_j < b_i;\\ (b_i+b_i^0-\sum_{j=1}^{n} a_{ij}x_j)/b_i^0, & b_i \le \sum_{j=1}^{n} a_{ij}x_j \le b_i+b_i^0, \ i=\overline{1,m};\\ 0, & \sum_{j=1}^{n} a_{ij}x_j > b_i+b_i^0.\end{cases} \tag{12}$$

Based on the definition of the Bellman-Zadeh fuzzy solution [21], the fuzzy linear programming problem (5)–(6) can be written in the form of an optimization problem of the following form: find the value of the parameter $\lambda \in [0,1]$, that is the solution of the linear programming problem

$$\max_x \lambda$$

with constraints

$$\mu_{\widetilde{G}}(x) \ge \lambda; \quad \mu_{\widetilde{F}_i}(x) \ge \lambda; \quad x \ge 0. \ \ i=\overline{1,m}. \tag{13}$$

Substituting (11) and (12) into (13), we write the final form of the optimization problem

$$\max_x \lambda$$

with constraints

$$\lambda(U-L)-\sum_{j=1}^{n} c_j x_j + L \le 0;$$

$$\sum_{j=1}^{n} a_{ij}x_j \le b_i + b_i^0 - \lambda b_i^0, \quad i=\overline{1,m}; \quad x \ge 0, \quad 0 \le \lambda \le 1. \tag{14}$$

This problem is a classical linear programming problem, for finding solutions to which any variant of the simplex method can be applied.

Let us assume that in the formulation of the problem of distributing the power of data transmission channels, the current values of the throughput of communication channels of each subscriber *k,* C_k, $k=\overline{1,N_3}$, are known, and the values of C_k^+, $k=\overline{1,N_3}$, determine the values of the bandwidths that are planned by users as a result of updating communication equipment. Obviously, it is possible to fully satisfy the expansion of the bandwidth of subscriber channels only under the condition $\sum_{j=1}^{N_2} B_j^+ \ge \sum_{k=1}^{N_3} C_k^+$.

Formally, the statement of problem can be written as

$$\max t_1; \ \max t_2; \ \ldots \max t_{N_3}, \tag{15}$$

with the following constraints

$$t_k \in \operatorname{supp} \tilde{t}_k = [C_k, C_k^+], \ \ k = \overline{1, N_3}; \ \ \sum_{k=1}^{N_3} t_k \le \sum_{i=1}^{N_1} A_i^+;$$

$$t_k \le B_j^+, \ \ j = \overline{1, N_2}, \ \ k = \overline{1, N_3}; \ \ \sum_{j=1}^{N_2} B_j^+ \le \sum_{i=1}^{N_1} A_i^+ \le \sum_{k=1}^{N_3} C_k^+ . \tag{16}$$

We will assume that the capacities of communication channels available to users satisfy the conditions $\sum_{k=1}^{N_3} C_k \le \sum_{k=1}^{N_3} t_k \le \sum_{i=1}^{N_1} A_i^+$, and the values of the possible expansion of the channel capacity are determined by right-hand fuzzy triangular numbers in the form (C_k, C_k, C_k^+), $k = \overline{1, N_3}$, with linear membership functions (2).

This problem is a multiobjective optimization problem. To solve it, methods are used that allow finding a compromise (effective) solution by reducing the problem to a single-criterion one in the form of a convolution of criteria or to a sequence of single-criteria optimization problems [22]. In the case of fuzzy constraints, each such problem can be reduced to an optimization problem of the form (13) or (14) with subsequent solution by the method proposed above.

Taking into account the specifics of the obtained problem, the most rational method is the sequential introduction of constraints [22]. A characteristic feature of this method, which makes it possible to use it to find an effective solution, is the sequential (at each step) introduction of constraints on the width of the communication channel, at which unsatisfactory values of the criteria are achieved.

Following the search methodology, at each algorithm's step $p = 1, 2, \ldots$, an "ideal assessment" $t^{*(p)} = (t_1^{*(p)}, t_2^{*(p)}, \ldots, t_{N_3}^{*(p)})$, $p = 1, 2, \ldots$, is formed, where $t_k^{*(p)}$, $k = \overline{1, N_3}$, are the optimal values of each of the criteria (19) $\max t_k$, $k = \overline{1, N_3}$, on a given range of acceptable values G_p, $G_1 = \{t_k = C_k^+; k = \overline{1, N_3}\}$, $G_{p+1} = \{t_k \in G_p; k = \overline{1, N_3} \mid t_s \ge \xi_s\}$, $s \in \{1, 2, \ldots, N_3\}$ is the number of the criterion, the value of which is the least consistent with the compromise solution. It is clarified to what level ξ_s the value of this criterion should be changed, and a search for a new solution is performed, taking into account the additional constraint.

This method allows solving the problem of efficient distribution of channel capacities, taking into account fuzzy constraints on consumption volumes, however, to use it at each step, it is necessary to evaluate the compliance of the current solution with a certain "ideal" solution, which, as a rule, is formed with the participation of an expert. In addition, the solution procedure turns out to be cumbersome, leading to the multiple solution of optimization problems of the form (7)–(10) and the construction of a Bellman-Zade fuzzy solution (14).

Additionally it is easy to formalize this process by applying the back tracking solution search procedure [23].

From the condition of the problem of optimizing the distribution of channel powers, taking into account fuzzy constraints on consumption volumes (15)–(16), it follows that

$$\sum_{k=1}^{N_3} C_k^+ \ge \sum_{j=1}^{N_2} B_j^+ \ge \sum_{k=1}^{N_3} C_k .$$

Obviously, in this case, it is impossible to allocate the maximum expected power of communication channels to all subscribers. We will look for a solution on rational distribution based on the scheme of the back tracking algorithm.

Algorithm.

Step 0. Without loss of generality, we will assume that the order of users is ordered in non-increasing order of the planned capacities of communication channels. We put the required values in the initial solution $t_k = C_k^+$, $k = \overline{1, N_3}$.

Step $s = 1, 2, \ldots$ *s=1,2,...* We check the fulfillment of condition

$$\sum_{k=1}^{N_3} t_k \le \sum_{j=1}^{N_2} B_j^+ . \tag{17}$$

If inequality (17) is satisfied, the algorithm terminates, otherwise:

a) determine the q, $q \in [1, N_3]$, largest (first of N_3) values t_k, $k = \overline{1, N_3}$;

b) decrease the values t_k, $k = \overline{1, q}$, by $\Delta t > 0$: $t_k = t_k - \Delta t$, $k = \overline{1, q}$.

Obviously, the total demand in this case decreases.

Change $s = s + 1$ and move on to the next step.

RESULTS OF COMPUTATIONAL EXPERIMENTS

The algorithm proposed above for finding a solution in the problem of rational distribution of the power of communication channels, taking into account fuzzy constraints on consumption volumes (15)–(16), was used to calculate the values of throughput resources in a network with 1 Internet provider, 2 (3, 4) routers (communication servers) and 17 end users (collective subscribers).

The bandwidth of user connections to communication servers was initially 350, 250, 250, 245, 180, 180, 165, 165, 160, 145, 140, 140, 140, 120, 110, 80, 80 Mb/s (total capacity 2900 Mb/s). In order to expand consumer traffic, it is proposed to upgrade equipment in the form of a possible increase in the number of servers or/and increase their capacity. The bandwidth of the communication channel with the provider remains constant and equals 10 Gb/s. The total throughput capacity of communication servers after the upgrade is planned to be 3 Gb/s.

To determine the rational distribution of the size of communication channels, consumers were asked to determine the required size of connections to communication servers. Based on the given amount of traffic, it was planned to use 2, 3 or 4 servers with a total capacity of 3 Gb/s.

Computational experiments on the efficient distribution of the power of communication channels were carried out using the above algorithm for the classical solution of optimization problems with fuzzy constraints on consumption levels (fuzzy approach) and the algorithm using the backtracking approach. In the latter case, both a consistent uniform decrease in consumer requests by the value $\Delta t > 0$ (app1) and a proportional decrease in the values of requests were applied, taking into account the required volumes of traffic increase (app2).

The results of the numerical experiments performed are shown in Table.

Table 1. The results of numerical experiments on the efficient distribution of the power of communication channels

Consumers																	
p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}	Sum
Init Power, Mb/s (*Max Sum Power*=2900 Mb/s)																	
350	250	250	245	180	180	165	165	160	145	140	140	140	120	110	80	110	2900
Plan Power, Mb/s (*Max Sum Power*=3000 Mb/s)																	
370	275	275	260	195	185	180	175	165	155	150	150	145	125	115	90	115	3100
Results for K communication servers, Mb/s																	
Approuch: ***app1***																	
*CommunicationPower**K*=1500×2																	
363	268	268	253	188	183	173	168	158	148	143	143	138	118	108	85	108	2995
*CommunicationPower**K*=1000×3																	
359	264	264	249	184	179	169	164	154	144	139	139	134	114	105	83	105	2934
*CommunicationPower**K*=750×4																	
357	262	262	247	182	177	167	162	152	142	137	137	132	112	107	89	107	2914
Approuch: ***app2***																	
*CommunicationPower**K*=1500×2																	
354	254	254	254	184	184	174	169	164	149	149	149	144	124	114	89	114	2999
*CommunicationPower**K*=1000×3																	
352	252	252	252	182	182	172	167	162	147	147	147	142	122	112	87	112	2967
*CommunicationPower**K*=750×4																	
350	250	250	250	180	180	170	165	160	145	145	145	140	120	110	85	110	2935
Approuch: ***fuzzy***																	
*CommunicationPower**K*=1500×2																	
361	266	266	251	186	181	171	166	160	150	146	144	141	121	110	83	110	2985
*CommunicationPower**K*=1000×3																	
362	267	267	252	187	182	173	167	160	150	147	142	140	120	110	82	110	2989
*CommunicationPower**K*=750×4																	
355	260	260	245	180	175	165	160	150	150	145	140	135	118	108	79	108	2900

As follows from the results obtained, the application of the proposed algorithm made it possible to obtain the most efficient (close to optimal) solutions in the considered distribution problem for a configuration with two communication servers with a maximum bandwidth of 1500 Mb/s. The best solution to the problem using the method of efficient channel power distribution, taking into account fuzzy constraints, was obtained for the connection option with 3 routers. At the same time, it slightly differs from the solution with two servers, which suggests that the best option in the considered distribution problem is the variant with two communication servers. It should also be noted that the solution based on the algorithm using the return scheme does not require significant computational resources, which allows us to speak about the constructiveness of the method. The resulting solution was used as the basis for the technical modernization of equipment to ensure the operation of network subscribers.

DISCUSSION AND CONCLUSIONS

Several remarks should be noted. First, the amount Δt of change in the power of communication channels in the backtracking algorithm, which is set at the beginning of the work, depends on the values of the optimized data transfer volumes and affects the rate of convergence of the algorithm. The choice of small values Δt leads to a more accurate rational distribution of powers, but slows down convergence. Otherwise, for large values Δt, the solution is reached faster, but its quality in terms of the obtained volumes, as a rule, turns out to be lower.

In addition, in the proposed version of the algorithm, a rational solution is sought at the expense of the most demanding subscribers in terms of volume. Obviously, the search procedure can be restructured to use other similar principles or to evenly distribute the redundancy of the total traffic request among all network users.

The problem of optimal power distribution of communication channels in information-computer networks with a three-level architecture is considered. Approaches for its solution are studied, the problem statement with fuzzy constraints on the consumption volumes of end users is considered. A fuzzy optimization problem is formulated, which allows taking into account the interval specified volumes for the connection values. A variant of solving fuzzy optimization problems in the case of using fuzzy numbers is proposed. A multi-objective problem of efficient power distribution of communication channels with fuzzy constraints is formulated. A variant of the algorithm with a return is proposed, which allows solving the obtained problem. The approach is illustrated by a number of numerical examples of a problem with a given number of end users and different allowable bandwidths of communication servers.

The results obtained were analyzed, which made it possible to make a decision on the method of upgrading the communication equipment. The proposed approach based on the method using the return scheme turned out to be a constructive way to solve the problem considered in the article.

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ПРО ДЕЯКІ МЕТОДИ РОЗВ'ЯЗАННЯ ЗАДАЧІ РОЗПОДІЛУ ПОТУЖНОСТІ КАНАЛІВ ПЕРЕДАВАННЯ ДАНИХ З УРАХУВАННЯМ НЕЧІТКИХ ОБМЕЖЕНЬ НА ОБСЯГИ СПОЖИВАННЯ / Є.В. Івохін, Л.Т. Аджубей, П.Р. Ваврик, М.Ф. Махно

Анотація. Розглянуто математичну постановку задачі оптимального розподілу потужностей каналів передавання даних в інформаційно-комп'ютерних мережах з трирівневою архітектурою та нечіткими обмеженнями на обсяги споживання. Розроблено ефективний алгоритм розв'язання задачі, особливістю якої є неможливість забезпечувати запити кінцевого споживача за рахунок ресурсів різних постачальників. Розглянуто стандартний метод розв'язання на основі нечіткої оптимізаційної задачі математичного програмування. Запропоновано конструктивний варіант пошуку розв'язку на основі методу з поверненням. Проведено обчислювані експерименти. Розроблено підхід, використаний для визначення оптимальної конфігурації трирівневої інформаційно-комп'ютерної мережі із заданою кількістю комунікаційних серверів.

Ключові слова: передавання даних, розподіл потужності, нечіткі обмеження, оптимальний розв'язок, алгоритм з поверненням.