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RESEARCHES AND APPLICATIONS OF THE COMBINATORIAL CONFIGURATIONS FOR INNOVATIVE DEVICES AND PROCESS ENGINEERING[1]

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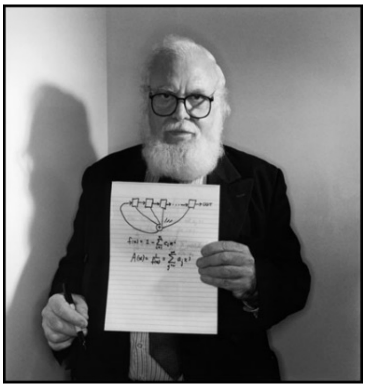
Abstract. This paper is devoted to the memory of Solomon Wolf Golomb (1932–2016) — a famous American mathematician, engineer, and professor of electrical engineering. He was interested in developing techniques for improving the quality indices of engineering devices and systems with respect to performance reliability, transmission speed, positioning precision, and resolving ability based on novel combinatorial configurations. In 1996 S. Golomb have supported the project “Researches and Applications of the Combinatorial Configurations for Innovative Devices and Process Engineering” as a scientific collaboration with the Former Soviet Union (FSU) research team from Lviv Polytechnic State University (Ukraine) under the Cooperative Grant Program by CRDF (U.S.). The underlying project to be edited by S. Golomb is presented, and short information on the development of the researches and applications of optimized systems with ring structure given.



Keywords: project, combinatorial configuration, Ideal Ring Bundle, optimization, model, scientific school.

INTRODUCTION

Solomon Wolf Golomb was a world-famous American scientist, specialist in the field of telecommunications, digital electronics, cellular automata and information theory. He was a world leader in the application of combinatorial mathematics in coding theory and radiophysics. Golomb was the inventor of Golomb coding, a form of entropy coding, the Golomb rulers used in astronomy and data encryption, also named after him, as was one of the main methods for generating Costas arrays, the Lempel–Golomb generation method.

S. Golomb developed the idea of using the advantages of multi-bit shift registers with a balanced number of 0 and 1, or 00, 01, 10, 11, revealing in them the absence of autocorrelation, which made it possible to improve encoding systems –

[1]This material is based upon project of the Lviv State Polytechnic University (Ukraine) to cooperate with the University of Southern California under of the U.S. Civilian Research & Development Foundation (CRDF): Researches and Applications of the Combinatorial Configurations for Innovative Devices and Process Engineering // *CRDF Cooperative Grants Program*, Los Angeles, CA 90089-2565, US, March 5, 1996, 10 p.

decoding signals with correction of errors using sequences generated by shift registers. S. Golomb used versions of these sequences (Reed-Solomon codes) to encode video images of Mars, in CDMA cell phones (Codé Division Multiple Access) with multiple access and code separation of communication channels. In 1956, he joined Glenn L. Martin Company, which later became a defense contractor [1]. This explains all his further scientific activities under conditions of strict secrecy, developing military and space communications. Continuing to study sequences on shift registers to improve radio control systems for missiles, S. Golomb with the help of the 85-foot Golstone radio antenna located in the Mojave Desert, specified data on the distances Earth – Venus and the Earth – the Sun. The sequences generated by the sequence shift registers made it possible to clarify the distance to Venus using a radar system. In 1985, Golomb received the Shannon Prize of the IEEE Society of Information Theory, and later the Medal of the U.S. National Security Agency. He was also the winner of the Lomonosov Medal of the Russian Academy of Sciences and the Kapitsa Medal of the Russian Academy of Natural Sciences. In 2000, Golomb was awarded the Richard W. Hemming IEEE Medal for his exceptional contributions to information science and systems engineering. In 2016, he was awarded the Benjamin Franklin Medal in Electrical Engineering for his pioneering work in space communications and digital signal processing, secure data forwarding, improving methods for deciphering cryptographic texts, rocket guidance systems, cellular communications, radars, sonar, GPS [1].

In 1996, S. Golomb have supported the proposal of the Lviv State Polytechnic University (Ukraine) to cooperate with the University of Southern California on the project "Researches and Applications of the Combinatorial Configurations for Innovative Devices and Process Engineering", which was sent to the U.S. Civilian Research & Development Foundation (CRDF). This non-profit organization was founded in 1995 by the National Science Foundation of the United States in accordance with the decision of the U.S. Congress in order to promote international scientific and technical cooperation with the provision of grants, technical resources, training for scientists and researchers. S. Golomb has edited the text of the project, motivating the advantages of the proposed combinatorial configurations with a ring topological structure over chain sequences.

NARRATIVE PROJECT

Research and Applications of Combinatorial Configurations for Innovative Devices and Process Engineering

- The objectives of the proposed project are as follows:

1. Research into the underlying mathematical principles relating to the optimal placement of structural elements in spatially or temporally distributed systems, including the appropriate algebraic constructions based on cyclic groups in extensions of Galois field; development of the scientific basis for technologically optimum distributed systems theory; and the generalization of these methods and results to the improvement and optimization of a larger class of technological systems.

2. Experimental verification of the effectiveness of this new methodology, as it affects the whole range of new high-performance devices, systems, or technolo-

gies to which it can be applied, including applications to coded design of signals for communications and radar, positioning of elements in an antenna array, and other areas to which the mathematical apparatus of contemporary combinatorial theory can be applied.

- Expected results of the completed project:

1. Mathematical results – development of new algebraic results and techniques, based on the idea of "perfect" combinatorial constructions, and expanding the applicability of cyclic group relationships in Galois fields, and a variety of results in number theory.

2. Physical results – a better understanding of the role of geometric structure in the behavior of natural and man-made objects.

3. Technological results – the development of new directions in fundamental and applied research in systems engineering, for improving such quality indices as reliability, precision, speed, resolving ability, and functionality, using innovative methodologies based on combinatorial techniques, with direct applications to radio- and electronic engineering, radio astronomy, applied physics, and other engineering areas.

"Ideal Ordered Chain" Combinatorial Constructions. The "ordered chain" approach to the study of elements and events is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a distributed technological system.

Sums on ordered-chain numbers. Let us calculate all S_n sums of the terms in the numerical *n*-stage chain sequence of distinct positive integers $C_n = \{K_1, K_2, \ldots, K_n\}$, where we require all terms in each sum to be consecutive elements of the sequence. Clearly the maximum such sum is the sum $K_1 + K_2 + \ldots + K_n = T$ of all *n* elements; and the maximum number of distinct sums is

$$S_n = 1 + 2 + \ldots + n = n(n+1)/2\,. \tag{1}$$

If we regard the chain sequence C_n as being *cyclic*, so that K_n is followed by K_1, we call this a ring sequence. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to $n-1$. In addition, there is the sum T of all n terms, which is the same independent of the starting point. Hence the maximum number of distinct sums $S(n)$ of consecutive terms of the ring sequence is given by

$$S(n) = n(n-1) + 1\,. \tag{2}$$

Comparing the equations (1) and (2), we see that the number of sums $S(n)$ for consecutive terms in the ring topology is nearly double the number of sums S_n in the daisy-chain topology, for the same sequence C_n of n terms.

Ideal Numerical Rings. An *n*-stage ring sequence $C_n = \{K_1, K_2, \ldots, K_n\}$ of natural numbers for which the set of all $S(n)$ circular sums consists of the numbers from 1 to $S(n) = n(n-1) + 1$ (each number occurring exactly once) is called an “Ideal Numerical Ring” (INR). Here is an example of an IRN with *n*=5 and $S(n)=21$, namely $\{1,3,10,2,5\}$. To see this, we observe:

$1=1$	$6=5+1$	$11=2+5+1+3$	$16=1+3+10+2$
$2=2$	$7=2+5$	$12=10+2$	$17=10+2+5$
$3=3$	$8=2+5+1$	$13=3+10$	$18=10+2+5+1$
$4=1+3$	$9=5+1+3$	$14=1+3+10$	$19=5+1+3+10$
$5=5$	$10=10$	$15=3=10=2$	$20=3+10+2+5$
			$21=1+3+10+2+5$

Note that if we allow summing over more than one complete revolution around the ring, we can obtain all positive integers as such sums. Thus:

$$22=1+3+10+2+5+1,\quad 23=2+5+1+3+10+2\text{, etc.}$$

Next, we consider a more general type of INR, where the $S(n)$ ring-sums of consecutive terms give us each integer value from 1 to M, for some integer M, exactly R times, as well as the value of $M+1$ (the sum of all n numbers) exactly once. Here we see that:

$$M=n(n-1)/R\,.$$

An example with $n=4$ and $R=2$, so that $M=6$, is the ring sequence (1, 1, 2, 3), for which the sums of consecutive terms are:

1, 1, 2, 3;			
$1+1=2,$	$1+2=3,$	$2+3=5,$	$3+1=4,$
$1+1+2=4,$	$1+2+3=6,$	$2+3+1=6,$	$3+1+1=5,$
$1+1+2+3=7.$			

We see that each “circular sum” from 1 to 6 occurs exactly twice $(R=2)$. We say that this INR has the parameters $n=4$, $R=2$.

- The individual competence of the FSU research team:

1. Theoretical research on the ideal configurations named Ideal Numerical Bundles (INB), in particular, Ideal Numerical Rings (INR), as modifications of certain combinatorial block-designs (existence, enumeration, classification).
2. Cyclic difference sets and some properties of Galois Field cyclic groups.
3. Construction of BIB designs, using INRs and Golomb rulers, and the reverse.
4. Software for construction of BIB designs using INRs.
5. Compiling a general catalogue of INRs and Golomb rulers.
6. Circuit design for the high performance technology of information coding and modulation.
7. Applied research and engineering design of concrete innovative devices and technologies based on combinatorial theory.
8. INBs and some problems of harmony and optimization of systems.

- The individual competence of the US research team:

1. Theoretical research on Golomb rulers and their relationships with difference (existence, enumeration, classification).
2. Construction of difference sets and Golomb rulers.
3. Compiling a general catalogue Golomb rulers.

4. Circuit design of high performance technology for coding of information and the design of communication signals.

5. Applied research and engineering design of innovative devices and technologies based on combinatorial theory.

6. Using combinatorial designs to obtain sequences with favorable correlation properties.

For carrying out the project, it is necessary to combine the achievements of Golomb ruler theory (US) and possibilities of Ideal Bundles theory (FSU) for extending the sphere of practical applications, with the aim of obtaining commercial products.

The FSU team developed the basis of the bundles theory as a new modification of combinatorial configurations on graphs involving Golomb ruler theory and proposed a new approach to the synthesis of technical devices and to engineering technology.

The US team developed the basis of Golomb ruler theory and proposed areas of their possible applications.

Equipment: 1) PC TSDX-4-120 540M, SVGA 0.28 LR NI 1M, 1, 2+1, 44; 2) Printer EPSON Stylus 800; 3) Telecommunications Services: Scanner HEWLETT PACKARD Jet-555, Fax-modem US Robotics Sportster; 4) Writing materials, floppy disk.

The FSU and U.S. co-investigators the project implementation and assess progress ar regular intervals (monthly) by using Fax and E-mail.

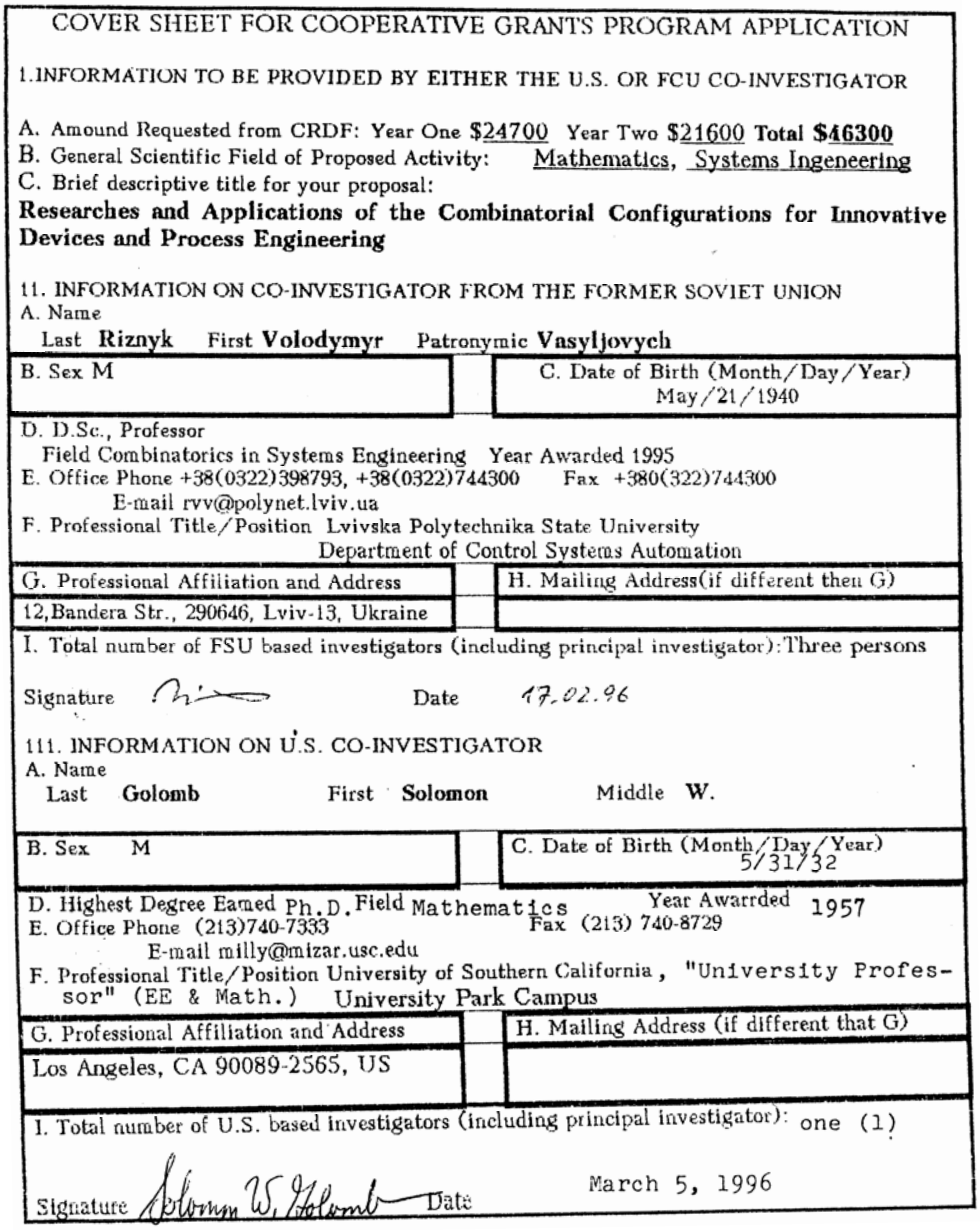
PLANNED STEPS

Title of stages	Stage duration	Anticipated results
Theoretical research on Ideal Numerical Rings (INR)	6 mos	Conditions of INR existence will be determined
Research of properties of extended Galois Field cyclic groups	6 mos	Patterns in the distribution of cyclic group elements will be established
Construction of INRs and Ideal numbere ring configurations	6 mos	Catalog of INRs
Applied research and engineering design of specific innovative devices and technologies	6 mos	Creation of concrete innovative devices and technologies

CONCLUSION

The Ideal Numerical Bundles (INB)s provide, essentially, a new conceptual model of technical systems. Moreover, the optimization has been embedded in the underlying combinatorial models. The remarkable properties and structural perfection of one- and multidimensional INBs provide an ability to reproduce the maximum number of combinatorial varieties in the systems with a limited number of elements and bonds. The favorable qualities of the combinatorial structures provide many opportunities to apply them to numerous branches of science and advanced technology. A perfection, beauty and harmony exist not only in the abstract models but in the real world also.

Here is the print of the cover page of the project, worded by S. Golomb.



COVER SHEET FOR COOPERATIVE GRANTS PROGRAM APPLICATION

1.INFORMATION TO BE PROVIDED BY EITHER THE U.S. OR FCU CO-INVESTIGATOR

A. Amound Requested from CRDF: Year One $24700 Year Two $21600 **Total $46300**
B. General Scientific Field of Proposed Activity: Mathematics, Systems Ingeneering
C. Brief descriptive title for your proposal:
Researches and Applications of the Combinatorial Configurations for Innovative Devices and Process Engineering

11. INFORMATION ON CO-INVESTIGATOR FROM THE FORMER SOVIET UNION
A. Name
Last **Riznyk** First **Volodymyr** Patronymic **Vasyljovych**

B. Sex M
C. Date of Birth (Month/Day/Year) May/21/1940

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H. Mailing Address(if different then G)

I. Total number of FSU based investigators (including principal investigator): Three persons

Signature Date 17.02.96

111. INFORMATION ON U.S. CO-INVESTIGATOR
A. Name
Last **Golomb** First **Solomon** Middle **W.**

B. Sex M
C. Date of Birth (Month/Day/Year) 5/31/32

D. Highest Degree Eamed Ph.D. Field Mathematics Year Awarrded 1957
E. Office Phone (213)740-7333 Fax (213) 740-8729
E-mail milly@mizar.usc.edu
F. Professional Title/Position University of Southern California, "University Professor" (EE & Math.) University Park Campus

G. Professional Affiliation and Address
Los Angeles, CA 90089-2565, US
H. Mailing Address (if different that G)

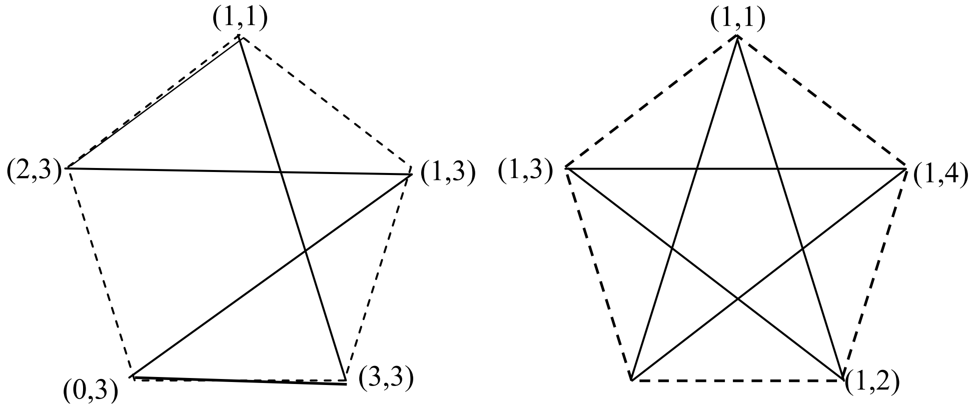
I. Total number of U.S. based investigators (including principal investigator): one (1)

Signature Solomon W. Golomb Date March 5, 1996

Note, S. Golomb personally signed the date of his birthday 31.05.1932 (not 30.05.1932, as indicated in the Wikipedia [1]).

AFTERWORD

In mathematics, a Golomb ruler is a set of marks at integer positions along a ruler such that no two pairs of marks are the same distance apart. The Golomb ruler was named for Solomon W. Golomb. There is no requirement that a Golomb ruler be able to measure all distances up to its length, but if it does, it is called a perfect Golomb ruler. It has been proved that no perfect Golomb ruler exists for five or more marks [2]. Unlike of them, an infinite quantity of one- and multidimensional IRBs exist, and the more long is an n-stage of IRB, the more invariants we can see by a majority. For example, the two variants of one-dimensional IRBs $\{1,3,10,2,5\}$ and $\{1,1,2,3,4\}$ exist for $n=5$ [3], while exactly the 1717 ones with $R=1$ for $n=102$ [4, p.163]. Of very interest are two-dimensional IRBs, which properties hold for the same set of the rings in varieties permutations of terms in the set, e.g. set of two-dimensional five-stages ($n=5$) ring sequences {(1,1),(1,0),(1,4),(1,3),(1,2)} and {(1,1),(1,3),(1,0),(1,2), (1,4)}, called "Gloria to Ukraine Stars" [5]. Examples of two paired pentagonal (n=5) such IRBs shown below.



Two paired pentagonal (n=5) "Gloria to Ukraine Stars"

It is shown that the star{(1,1), (1,3), (3,3), (0,3),(2,3)} to be marked with dash edges, and the {(1,1), (2,3), (1,3), (0,3), (3,3)} – solid ones is the paired star (the outside star). The star {(1,1), (1,4), (1,2), (1,0), (1,3)} marked with dash edges, and the star {(1,1), (1,0), (1,4), (1,3), (1,2)} – solid ones is paired star also (the inner star). We have found numerous families of the two- and multidimensional stars, originated from the remarkable geometric properties and creative harmony of the real word [6].

S. Golomb's aptitude allowed him to see in the proposed project not only the "ideal ordered chain" combinatorial configurations but also the "ideal rings", which provide novel opportunities for the development of a new direction of fundamental and applied research in systems engineering with the direct use of one- and multidimensional combinatorial configurations of ring topology in radio electronics, communication, and numerous fields of advanced technologies. He not only approved our project, but also edited and substantiated the prospects for the implementation and further development of fundamental and applied research of combinatorial structures with ring topology, placing the right emphasis on the advantages of the latter over systems with a chain structure. Our collaboration was started in August 1996 from the report "Combinatorial Sequencing Theory for Optimisation of Signal Data Vectors Converting and Signal Processing" [7]. However, shortly we have got refusal letters from the CRDF in financial supporting the project.

Brief data on the scientific school of combinatorics at the Lviv Polytechnic National University

The scientific school of combinatorics at Lviv Polytechnic National University became known due to the fundamental and applied research of the "intelligent" two- and multidimensional combinatorial configurations prospected from the real world law of "elegant" deviding rotational symmetry into two complementary asymmetric parts. Each of them is an IRB with appropriate information parameters which being interrelated by this symmetry [6]. Design of systems with the "ideal" ring structures originated from solution of the engineering problem for expanding the range of time delays in transient analyzer on capacitor storage for researching dynamic processes in power electric stations on analog computers in 1968. One of the first publications for the perfect rings is connected with designing optimized memory devices [8]. The mathematical problem was to fix four switches on a moving rotor with different relative angular shifts of the ring switch, with the possibility of obtaining the widest possible range of time delays on a set of combination options for selecting the corresponding pair of swithes, one of them should bring the flowing voltage value to the memorizing element, and the second – to read the same value with a delay in time after turning the rotor of the switch to the appropriate angle [9]. Currently, we have a lot of patents, based on the idea of "perfect" one- and multidimensional IRBs.

The remarkable properties both optimal Golomb rulers and IRBs discover many opportunities to apply them to numerous branches of science and advanced technology, including vector information technologies and communications, vector data coding and multidimensional signal processing. These properties embedded in the laws of real world penetrating rotational symmetry.

ACKNOWLEDGMENTS

Honoring the memory about the outstanding American scientist S. Golomb, the research group "Vector Data Informatics" of Lviv Polytechnic National University expresses gratitude to all those who contributed to the development of scientific cooperation between California and Lviv Universities in the field of research and applications of combinatorial optimization methodology for information and telecommunication technologies. I grateful also to my colleagues from Automated Control Systems Department of Lviv Polytechnic National University for their active participation in the R&S project "Designing Software for Vector Data Processing and Information Protection Based on Combinatorial Optimization", (No state registration 0113U001360).

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INFORMATION ON THE ARTICLE

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ДОСЛІДЖЕННЯ ТА ЗАСТОСУВАННЯ КОМБІНАТОРНИХ КОНФІГУРАЦІЙ ДЛЯ ІННОВАЦІЙНИХ ПРИСТРОЇВ І ТЕХНОЛОГІЙ / В.В. Різник

Анотація. Присвячено пам'яті Соломона Вольфа Ґоломба (1932–2016) — відомого американського математика, інженера, професора електротехніки, який зробив значний внесок у розвиток теорії лінійних регістрів зсуву і комбінаторної математики в теорії кодування та радіофізиці. Він цікавився розробленням методів підвищення якісних показників інженерних пристроїв і систем з погляду надійності, швидкості передавання, точності позиціонування і роздільної здатності на основі нових комбінаторних конфігурацій. У 1996 р. С. Ґоломб підтримав проєкт "Дослідження та застосування комбінаторних конфігурацій для інноваційних пристроїв та технологій" як наукову співпрацю дослідницької групи Державного університету «Львівська політехніка» (Україна) і Південно-каліфорнійського університету за спільною грантовою програмою Фонду цивільних досліджень та розвитку (США). Подано основний проєкт під редакцією С. Голомба, а також коротку інформацію про розвиток досліджень та застосування оптимізованих систем з кільцевою структурою.

Ключові слова: проєкт, комбінаторна конфігурація, ідеальна кільцева в'язанка, оптимізація, комбінаторна модель, наукова школа.