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DECENTRALIZED LEADER-FOLLOWING CONSENSUS CONTROL DESIGN FOR DISCRETE-TIME MULTI-AGENT SYSTEMS WITH SWITCHING TOPOLOGY

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Abstact. The problem of consensus control of linear discrete-time multi-agent systems (MASs) with switching topology is considered in the presence of a leader. The goal of consensus control is to bring the states of all agents to the leader state while providing stability for local agents, as well as the MAS as a whole. In contrast to the traditional approach, which uses the concept of an extended dynamic multi-agent system model and communication topology graph Laplacian, this paper proposes a decomposition approach, which provides a separate design of local controllers. The control law is chosen in the form of distributed feedback with discrete PID controllers. The problem of local controllers' design is reduced to a set of semidefinite programming problems using the method of invariant ellipsoids. Sufficient conditions for agents' stabilization and global consensus condition fulfillment are obtained using the linear matrix inequality technique. The availability of information about a finite set of possible configurations between agents allows us to design local controllers offline at the design stage. A numerical example demonstrates the effectiveness of the proposed approach.

Keywords: multi-agent system, consensus control, switching topology, PID controller, invariant ellipsoids method, linear matrix inequality, semidefinite programming problem.

INTRODUCTION

Recently, consensus control of multi-agent systems (MAS) with networked structures has attracted the great attention of many researchers from different fields of science and engineering [1; 2]. In the field of automatic control, the development of consensus control theory is stimulated, in particular, by the rapid development of unmanned mobile vehicles and ensuring their coordinated behavior in accordance with the common goal [3]. Similar problems also arise under the control of large-scale systems with networked structures, such as complex technological and automated production systems, supply and logistics chains, and energy and transport systems as well.

The main problem of consensus control of MAS is the design of a control law, which allows all agents to reach the agreed values of their state or output variables, using the information obtained from other agents. Here, the control law is constructed based on a consensus protocol [2; 4]. The protocol design assumes that the control for each local agent is formed on the basis of information about the deviations of the state or output vector of any agent from the corresponding vectors of neighbouring agents directly related to it. In this case, the corresponding control system also has a network structure wherein the rules for information exchange between each local agent and its directly connected neighbours are determined by the network topology.

A topology model usually describes the MAS structure as connections between agents in the form of a directed graph, in that the nodes of which correspond to the controlled local agents, where the graph edges describe the information transfer channels between them. Usually, a consensus protocol is taken in the form of a linear deviation feedback between local agent states or outputs and the weighted average vector of states or outputs of its immediate neighbours. In such a case, the control problem is reduced to finding a set of feedback gain matrices from the stability condition for both local controlled agents and the MAS as a whole, considering the relationship between them, as well as the condition of reaching a consensus.

Taking into account the peculiarities of practical problems of MAS control leads to the need to complicate the problems under consideration. In reality, due to breaks in communication channels, the topology of connections can be arbitrarily changed by switching between elements of a finite set of possible configurations, which leads to the need for consensus control design under switching topology conditions.

REVIEW AND ANALYSIS OF INFORMATION SOURCES

In the last years, consensus problem research has developed very rapidly and numerous results have been obtained concerning distributed consensus protocols for MAS design (see [5; 6] and references therein).

The usual approach to solving consensus control problems is based on a dynamic model of MAS with an extended state vector composed of the state vectors of local agents, thus constructing a model using the concept of the Laplacian of the communication topology graph [7]. At that, the Kronecker product of the dynamic matrices of local agents describes the matrix of the extended MAS model dynamics.

Efficient methods for studying the stability of such systems have been developed; for the synthesis of consensus control, modern methods for controllers' design in state space, including the methods of linear matrix inequalities (LMIs), are widely used. In [8], using the LMI technique, a new form of state-feedback consensus control based on the aggregate Laplacian is proposed and sufficient conditions of stabilization are established using the Lyapunov stability theory.

Early work in this area dealt mainly with the problem of ensuring consensus, represented in the form of balance ratios of an agent's state vectors so that all agents are driven to converge to a common state, determined by the consensus conditions. Further development of consensus control is associated with an additional condition of following the leader, which is considered an agent that imposes the desired behavior on others [5]. A number of works have been devoted to the consensus control problems in multi-agent systems with a leader, see links in [9].

A large number of works are devoted to the consensus control design problem under switching topology conditions; systematized results are given in [10]. In [11], it was shown that under certain assumptions, consensus can be achieved asymptotically under dynamically changing interaction topologies if the union of the collection of interaction graphs across some time intervals has a spanning tree frequently enough. Consensus of MAS in a continuous time domain under fixed and switching topology was studied in [12], where the dynamics of local and leader agents are considered linear; the design technique was based on Riccati inequality and Lyapunov inequality. These results have also been generalized to discrete-time systems. In [13], sufficient conditions for the solvability of consensus problems for discrete-time multi-agent systems with switching topology and time-varying delays have been presented. In [14], the consensus problem for MAS was also considered in the discrete-time domain, the topology of interactions between agents was assumed to be switched and undirected. In [15], the cooperative control problem of discrete-time multi-agent systems is discussed, bounded uncertain time delay and directed switching topology are considered, and sufficient conditions for asymptotic consensus of the system under directed switching topology are obtained. State-of-the-art survey on consensus control of network systems with switching network topologies, presented in [16] with emphasis on the relationships between the switching among different topology candidates and the networked control stability.

The architectural features of networked consensus control systems, combined with a natural desire to reduce the computational resources required to calculate controls in real time, stimulated an increase in interest in building distributed multi-agent systems with decentralized consensus control. From the viewpoint of the practical implementation of consensus control, a decentralized approach is of great interest, in which the local control for each agent is designed only using locally available information, so it requires less computational effort and is relatively more scalable with respect to the number of agents. A decentralized approach to asymptotic consensus control design for discrete MAS, where local agents exchange information only with their nearest neighbours is studied in detail by a number of researchers. In [17], for agents of the first order, a heuristic approach is proposed based on an analogy with the Vicsek model of the motion of a group of particles on a plane. In [18], an adaptive procedure for constructing control for each agent is proposed using only information from its neighbours in the network topology. The consensus problem for a multi-agent system with high-order linear dynamics and a fully decentralized consensus algorithm is proposed in [19], which allows each agent to reach the consensus value using only a finite number of steps of its past state information, but the solution is obtained under the condition that the system has a time-invariant topology. A fully decentralized algorithm that allows any agent to compute the consensus value of the whole network in a finite time using only the minimal number of successive values of its own history was proposed in [20], where was shown, that this minimal number of steps is related to a Jordan block decomposition of the network dynamics. However, this minimum number of steps is related to graph theoretical notions that can be directly computed from the Laplacian matrix of the graph and from the minimum external equitable partition. Decentralized event-triggered finite-time consensus control under a directed graph was investigated in [21], and an adaptive law is designed to counteract the effect of uncertainties and external disturbances.

The quality of consensus control in transients, which is especially important in leader-following tracking problems, can be significantly improved using more complex dynamic consensus protocols, in which control actions depend not only on the current but also on previous deviations. As such control laws, multivariable Proportional-Integral-Derivative (PID) controllers are widely used, which make it possible to significantly improve the quality of consensus control in comparison with static protocols.

Despite its well-known benefits, PID control is lightly addressed in the decentralized MAS control and they are mainly dealt with homogeneous MASs (see e.g. [22] and references therein). For instance, a robust PID consensus control strategy has been proposed in [23] for a system of linear high-order agents under the restrictive assumption of an undirected communication graph. A PD protocol is proposed in [24] to solve, instead, the problem of the average consensus under a fast arbitrarily switching topology for the case of first-order nonlinear homogeneous MASs with Lipschitz dynamics. To solve the leader tracking for uncertain high-order homogeneous MASs, robust PID protocols have been investigated [25]. Nevertheless, practical applications of the networked MASs require heterogeneous models due to the presence of mismatches and differences among the agents. In this context, only a few decentralized protocols aim to extend the PID-control advantages to the heterogeneous MAS framework. In [26] it was investigated the use of distributed PID actions to achieve consensus in networks of homogeneous and heterogeneous linear systems. The convergence of the strategy is proven for both cases using appropriate state transformations and Lyapunov functions. A multiplex proportional–integral approach for solving consensus problems in networks with heterogeneous node dynamics affected by constant disturbances was proposed in [27]. The proportional and integral actions are deployed on two different layers across the network, each with its own topology. Sufficient conditions for convergence are derived that depend upon the structure of the network, the parameters and topologies characterizing the control layers, and the node dynamics. Fully-distributed PID control strategy was proposed in [28], whose stability is analytically proven by exploiting the controller equations and the Static Output Feedback procedure adapted to the MASs framework.

In this paper, the problem of decentralized PID controller design for leader-following consensus control of networked heterogeneous MASs is considered under the assumption that sharing information between agents is carried out via switching communication topology. In this work, we use a distributed consensus protocol in the form of local dynamic feedback with a PID controller using the signal of deviation of local agent states from the weighted average states of its neighbours, with which the agent exchanges information in the current period. The synthesis of local PID controllers is based on the method of invariant ellipsoids. To analyze the stability of closed-loop controlled local agents, as well as the stability of the whole MAS, the second Lyapunov method and LMI technique were used. This allowed us to reduce a local controller problem design to a problem of semidefinite programming to find the optimal gain values for each agent by numerically solving the optimization problem. This, in turn, makes it possible to simplify the solution in comparison with other well-known approaches, for example, with the descriptor method [29].

In summary, the main features of the proposed approach and contributions of the work are:

- Within the framework of the principle of decentralized consensus control, a method of local controllers' design is proposed, which ensures the stability of individually controlled agents and the networked multi-agent systems as a whole under conditions of a switching communication network topology.
- Based on the sufficient conditions of local-controlled agents' stability and reaching consensus as well as leader-following tracking in LMI form, a computational procedure for optimizing the parameters of local controllers by solving a set of semidefinite programming problems is proposed; while controller parameters optimization is performed offline during the design phase.
- A decentralized optimal dynamic consensus control strategy with a recurrent form of the PID control law is proposed, which makes it possible to abandon the construction of an extended model of a closed local agent, which is usually used for discrete systems with PID controllers, which makes it possible to reduce the optimization problem dimension.
- Availability of information about a finite set of topology variations of connections between agents allows solving the problem of calculating the feedback gain matrix before the start of the control process. Thus, to determine the control action at each step of the MAS operation from the resulting set of solutions, the appropriate one is selected depending on which neighbours the agent exchange information in the current step.

PROBLEM FORMULATION

Consider a MAS represented by a network of N agents as a set of multivariable discrete-time linear dynamic systems, where each i-th agent is described by the difference equation

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k),\ i = 1,...,N, \quad (1)$$

where $k = 0,1,2,...$ is time instant; $x_i(k) \in \mathbf{R}^n$, $u_i(k) \in \mathbf{R}^m$ are state and control vectors of i-th agent at time k; A_i, B_i are constant matrices of appropriate dimensions such that the system (1) is controllable.

The connection topology of a networked MAS is described by an undirected graph $G = (V, E)$, where $V = \{1,...,N\}$ is a set of nodes (i.e., agents), and $E \subset V \times V$ is the set of edges. The presence of an edge (i, j) in the graph G means that agents i and j exchange information. $D = [d_{ij}]$ is the adjacency matrix of graph G and has dimension $N \times N$.

Since the topology of the connections between agents can change during the system dynamics, the graph G can switch at arbitrary time moments among a finite set $G_1, G_2, ..., G_K$, each of which is an undirected graph, containing a spanning tree. This means that the graph G will have the form G_1 during a certain time interval, then G_j, $j \in \{1,...,K\}$, and so on, moreover, switching occurs arbitrarily.

A set of agents achieves consensus if the agents' states satisfy the consensus condition

$$\lim_{k\to\infty} (x_i(k) - x_j(k)) = 0,\ i, j = 1,...,N. \quad (2)$$

The control law $u_i(k)$, hereafter the consensus protocol, solves the consensus control problem if all agents achieve consensus under this control.

We construct a distributed consensus protocol in the form of local feedback to a PID controller using the signal of deviation of local agent states from the weighted average states of its neighbours

$$u_i(k) = K_P\Sigma_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k)(x_i(k) - x_j(k)) + K_I\Sigma_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k)\sum_{l=0}^{k-1}(x_i(l) - x_j(l)) +$$

$$+ K_D\Sigma_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k)((x_i(k) - x_j(k)) - (x_i(k-1) - x_j(k-1))), \qquad (3)$$

where $\Sigma_i = 1 \Big/ \sum_{j=1, j\neq i}^{N} d_{ij}$; $K_P, K_I, K_D \in \mathbf{R}^{m\times n}$ are the gain matrices of the proportional, integral, and differential parts of the controller, respectively; $\delta_{ij}(k)$ is binary variable, the value of which determines whether information about the state of the j -th agent is available to the i -th agent at a time k :

$$\delta_{ij}(k) = \begin{cases} 0, & \text{agent } i \text{ does not receive information from agent } j, \\ 1, & \text{otherwise.} \end{cases}$$

The recurrent form of control law (3) is more convenient for practical implementation, so the current value of the control action is determined by its previous value and the correction

$$u_i(k) = u_i(k-1) + K_{0i}\Sigma_i \sum_{j=1, j\neq i}^{N} d_{ij}\delta_{ij}(k)(x_i(k) - x_j(k)) +$$

$$+ K_{1i}\Sigma_i \sum_{j=1, j\neq i}^{N} d_{ij}\delta_{ij}(k)(x_i(k-1) - x_j(k-1)) +$$

$$+ K_{2i}\Sigma_i \sum_{j=1, j\neq i}^{N} d_{ij}\delta_{ij}(k)(x_i(k-2) - x_j(k-2)), \qquad (4)$$

where $K_{0_i}, K_{1_i}, K_{2_i} \in \mathbf{R}^{m\times n}$ are feedback coefficient matrices.

Introduce block matrix $K_i = [K_{0_i}, K_{1_i}, K_{2_i}]$ and the composite vectors $v_{ij}(k) = \text{col}\{(x_i(k) - x_j(k)), (x_i(k-1) - x_j(k-1)), (x_i(k-2) - x_j(k-2))\}$, $i, j = 1,\ldots,N$, $j \neq i$. Then, the consensus protocol (4) takes the form

$$u_i(k) = u_i(k-1) + \Sigma_i K_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k) v_{ij}(k) . \qquad (5)$$

The control law for the agent, acting as the leader, differs from (5) by addition the deviation term between the leader's state and the set point x^*

$$u_i^{leader}(k) = u_i(k) + K_{0i}(x_i(k) - x^*) .$$

The model of a closed-loop local agent with the consensus protocol (5) will take the form with one-step control lag

$$x_i(k+1) = (A_{iC} + B_i K_i)\Sigma_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k) v_{ij}(k) + A_i \sum_{j=1, j\neq i}^{N} \delta_{ij}(k) x_j(k) + B_i u_i(k-1), \; (6)$$

where $A_{iC} = [A_i \mid 0_{n\times n} \mid 0_{n\times n}]$ is a block matrix, $0_{n\times m}$ is the null matrix of the corresponding dimension. For the agent that is a leader, equation (6) differs by presence of the term $B_i K_{0i}(x_i(k) - x^*)$.

For a linear closed-loop discrete MAS with local feedback described by equation (6) under the conditions of an arbitrarily switching topology described by a finite set of undirected connected graphs $G_1, G_2, ..., G_K$, the problem of decentralized consensus control is considered. Such a problem is reduced to the choice of feedback gain matrices $K_i, i = 1,...,N$ that ensure the stabilization of closed-loop local agents, the stability of the controlled MAS as a whole, as well as the fulfilment of the consensus conditions (2).

CONSENSUS CONTROL DESIGN

The approach based on the second Lyapunov method, which allows us to obtain sufficient stability conditions for closed-loop local subsystems, is used to calculate the gain matrices of local controllers. The development of the theory of linear matrix inequalities [30] makes it possible to apply a similar approach to the synthesis of consensus control of multi-agent systems.

The main idea of linear feedback controller design using LMI is as follows. The control goal is formulated as an inequality with respect to the quadratic Lyapunov function built on the solutions of the closed-loop system. The resulting inequality is reduced to the LMI form with respect to the unknown matrix of the controller parameters. The specified constraints are also reduced to the LMI form. A certain criterion of optimality is used and the corresponding convex optimization problem is solved numerically, because of which the optimal parameters of the controllers are determined.

The considered technique is implemented based on the invariant ellipsoid's method [31]. Consider the ellipsoid described by the equation

$$\varepsilon_i(Q_i) = \{x \in \mathbf{R}^n : \; x_i^{\mathrm{T}}(k) Q_i \, x_i(k) \le 1\}, \tag{7}$$

where $0 \prec Q_i \in \mathbf{R}^{n\times n}$ is ellipsoid matrix; $M \succ 0$ $(M \succeq 0)$ means that the matrix M is positive (nonnegative) definite.

The ellipsoid (7) is called state invariant for the system (6) if any trajectory of the system, having started in the ellipsoid, remains in it for any time moment $k \ge 0$.

The stabilization problem of system (6) comes down to calculating the block matrices of feedback gain $K_i, i = 1,...,N$ such that the consensus protocol (5) provides minimization of ellipsoid (7) by some optimality criterion. We choose as a criterion the length squares sum of the ellipsoid semiaxes which is equal to the trace of its matrix Q_i.

Consider a quadratic function constructed from the solutions of the system (6)

$$V_i(k) = x_i^{\mathrm{T}}(k) P_i x_i(k), \quad 0 \prec P_i = P_i^{\mathrm{T}} \in \mathbf{R}^{n\times n}. \tag{8}$$

It is well known that function (8) is a Lyapunov function for system (6) if the conditions are fulfilled:

(i) the function values are non-negative for any $x_i(k) \neq 0$;

(ii) the function values decrease monotonically over time.

If equality holds

$$Q_i = P_i^{-1}, \tag{9}$$

then the invariant ellipsoid (7) is the level set of Lyapunov function candidate (8).

It was shown in [38] that for a stable and controllable discrete-time dynamical system, the solution of the minimization problem by some criterion of quadratic Lyapunov function under the constraint specified by the Lyapunov inequality, is achieved by the solution of the Lyapunov equation. Thus, such an approach allows reducing the robust control design problems with respect to the described class of system topology uncertainty, to solve the problem of minimizing a linear function under constraints that can be represented in the form of linear matrix inequalities, that is, to solve a semidefinite programming problem.

The hypothesis on the basis of which the design problem of consensus control for MAS with switching topology in the presence of a leader is solved is that for any version of the topology described by a finite set of undirected graphs $G_1, G_2, ..., G_K$, each of which contains a spanning tree, the statements of the following theorem are fulfilled.

Theorem. If for linear stable discrete-time system (6) matrices $\hat{Q}_i$, $\hat{Y}_{0i}$, $\hat{Y}_{1i}$, $\hat{Y}_{2i}$ are obtained by solving the optimization problem

$$\operatorname{trace}(Q_i) \to \min \tag{10}$$

subject to

$$\left[\begin{array}{ccccccccc}
 & \multicolumn{4}{c}{\overbrace{\hphantom{A_iQ_i \quad A_iQ_i \quad \dots \quad A_iQ_i}}^{N-1}} & \multicolumn{3}{c}{\overbrace{\hphantom{\Delta_{ij}(A_{iQ}+B_iY_i) \quad \dots \quad \Delta_{ij}(A_{iQ}+B_iY_i)}}^{N-1}} & \\
Q_i & A_iQ_i & A_iQ_i & \dots & A_iQ_i & \Delta_{ij}(A_{iQ}+B_iY_i) & \dots & \Delta_{ij}(A_{iQ}+B_iY_i) & B_i \\
* & Q_i & 0_{n\times n} & \dots & 0_{n\times n} & 0_{n\times 3n} & \dots & 0_{n\times 3n} & 0_{n\times m} \\
* & * & 0_{n\times n} & \dots & 0_{n\times n} & 0_{n\times 3n} & \dots & 0_{n\times 3n} & 0_{n\times m} \\
\vdots & & \vdots & & & & \vdots & & \vdots \\
* & * & * & * & 0_{n\times n} & 0_{n\times 3n} & \dots & 0_{n\times 3n} & 0_{n\times m} \\
* & * & * & * & * & 0_{n\times 3n} & \dots & 0_{n\times 3n} & 0_{n\times m} \\
\vdots & & \vdots & & & & \vdots & & \vdots \\
* & * & * & * & * & * & * & 0_{n\times 3n} & 0_{n\times m} \\
* & * & * & * & * & * & * & * & 0_{m\times m}
\end{array}\right] \succeq 0 \tag{11}$$

on matrix variables $0 \prec Q_i \in \mathbf{R}^{n\times n}$, $Y_i = [Y_{0_i}, Y_{1_i}, Y_{2_i}]$, $Y_{0_i}, Y_{1_i}, Y_{2_i} \in \mathbf{R}^{m\times n}$, where $\Delta_{ij} = \Sigma_i \delta_{ij}$, $A_{iQ} = [A_i Q_i \mid 0_{n\times n} \mid 0_{n\times n}]$, «∗» denotes the symmetric terms in then inequality matrix, then:

(i) for any initial state $x_i(0) \in \varepsilon_i(\hat{Q}_i)$ closed-loop system (6) is asymptotically stable;

(ii) among all consensus protocols of the form (5), the protocol with gain matrices

$$K_{0i}=\hat{Y}_{0i}\,\hat{Q}^{-1},\quad K_{1i}=\hat{Y}_{1i}\,\hat{Q}^{-1},\quad K_{2i}=\hat{Y}_{2i}\,\hat{Q}^{-1} \tag{12}$$

delivers the minimum of the matrix trace criterion for the invariant ellipsoid (7) of the closed-loop system (6) and hence guaranties the fulfillment of consensus condition (2).

Proof. The first of the conditions, which are necessary for a candidate (8) to be a Lyapunov function for the system (6), is satisfied due to the positive definiteness of the matrix P_i. Hence, the consensus protocol (5) should ensure that the second property is satisfied.

We calculate the difference of the candidates in the Lyapunov function (8) with respect to k and require that the value of the function decrease over time

$$V_i(k+1)-V_i(k)=s_i^{\mathrm{T}}(k)M_i s_i(k)\le 0\,, \tag{13}$$

where $s_i(k)=\mathrm{col}\{x_i(k),\ldots,x_N(k),v_{ij}(k),\ldots,v_{iN}(k),u_i(k-1)\}\in\mathbf{R}^{4Nn-3n+m}$,

$$M_i=$$

$$=\begin{bmatrix}
A_i^{\mathrm{T}}P_iA_i-P_i & \overbrace{A_i^{\mathrm{T}}P_iA_i \quad \ldots \quad A_i^{\mathrm{T}}P_iA_i}^{N-1} & \overbrace{A_i^{\mathrm{T}}P_i\Delta_{ij}\Omega_i \quad \ldots \quad A_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i}^{N-1} & A_i^{\mathrm{T}}P_iB_i\\
* & A_i^{\mathrm{T}}P_iA_i \quad \ldots \quad A_i^{\mathrm{T}}P_iA_i & A_i^{\mathrm{T}}P_i\Delta_{ij}\Omega_i \quad \ldots \quad A_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i & A_i^{\mathrm{T}}P_iB_i\\
\vdots & \vdots & \vdots & \vdots\\
* & * \quad * \quad A_i^{\mathrm{T}}P_i\,A_i & A_i^{\mathrm{T}}P_i\Delta_{ij}\Omega_i \quad \ldots \quad A_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i & A_i^{\mathrm{T}}P_iB_i\\
* & * \quad * \quad * & \Delta_{ij}\Omega_i^{\mathrm{T}}P_i\Delta_{ij}\Omega_i \quad \ldots \quad \Delta_{ij}\Omega_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i & \Delta_{ij}\Omega_i^{\mathrm{T}}P_iB_i\\
* & * \quad * \quad * & * \quad \ldots \quad \Delta_{ij}\Omega_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i & \Delta_{ij}\Omega_i^{\mathrm{T}}P_iB_i\\
\vdots & \vdots & \vdots & \vdots\\
* & * \quad * \quad * & * \quad * \quad \Delta_{iN}\Omega_i^{\mathrm{T}}P_i\Delta_{iN}\Omega_i & \Delta_{iN}\Omega_i^{\mathrm{T}}P_iB_i\\
* & * \quad * \quad * & * \quad * \quad * & B_i^{\mathrm{T}}P_iB_i
\end{bmatrix},$$

$$\Omega_i=A_{iC}+B_iK_i\,.$$

Inequality (13) is equivalent to the matrix inequality $M_i\preceq 0$. Let us represent the inequality matrix in the form

$$M_i=\begin{bmatrix} A_i^{\mathrm{T}}\\ \vdots\\ A_i^{\mathrm{T}}\\ \Delta_{ij}\Omega_i^{\mathrm{T}}\\ \vdots\\ \Delta_{iN}\Omega_i^{\mathrm{T}}\\ B_i^{\mathrm{T}}\end{bmatrix}\cdot P_i\cdot\begin{bmatrix}A_i & \ldots & A_i & \Delta_{ij}\Omega_i & \ldots & \Delta_{iN}\Omega_i & B_i\end{bmatrix}+$$

$$+\begin{bmatrix}-P_i & 0_{n\times(4(N-1)n+m)}\\ * & 0_{(4(N-1)n+m)\times(4(N-1)n+m)}\end{bmatrix}.$$

Using the Schur complement [32], inequality takes the form

$$\begin{bmatrix} -P_i^{-1} & \overbrace{A_i \quad A_i \quad \cdots \quad A_i}^{N-1} & \overbrace{\Delta_{ij}\Omega_i \quad \cdots \quad \Delta_{ij}\Omega_i}^{N-1} & B_i \\ * & -P_i \quad 0_{n\times n} \quad \cdots \quad 0_{n\times n} & 0_{n\times 3n} \quad \cdots \quad 0_{n\times 3n} & 0_{n\times m} \\ * & * \quad 0_{n\times n} \quad \cdots \quad 0_{n\times n} & 0_{n\times 3n} \quad \cdots \quad 0_{n\times 3n} & 0_{n\times m} \\ \vdots & \vdots & \vdots & \vdots \\ * & * \quad * \quad * \quad 0_{n\times n} & 0_{n\times 3n} \quad \cdots \quad 0_{n\times 3n} & 0_{n\times m} \\ * & * \quad * \quad * \quad * & 0_{n\times 3n} \quad \cdots \quad 0_{n\times 3n} & 0_{n\times m} \\ \vdots & \vdots & \vdots & \vdots \\ * & * \quad * \quad * \quad * & * \quad * \quad 0_{n\times 3n} & 0_{n\times m} \\ * & * \quad * \quad * \quad * & * \quad * \quad * & 0_{m\times m} \end{bmatrix} \preceq 0 .$$

By multiplying the left and right parts of the inequality by -1, performing substitution (9) and applying a congruent transformation to the inequality matrix with $\mathrm{block\,diag}\{I_n, \underbrace{Q_i, ..., Q_i}_{4(N-1)}, I_m\}$, where I_n is identity matrix of the corresponding dimension, we obtain the matrix inequality, which is nonlinear with respect to the matrix variables Q_i and K_i. We introduce matrix variables $Y_{0i} = K_{0i}\, Q_i$, $Y_{1i} = K_{1i}\, Q_i$, $Y_{2i} = K_{2i}\, Q_i$. Whence, by virtue of $Q_i \succ 0$ the matrices K_{0i}, K_{1i}, K_{2i} recovers uniquely in accordance with (12). Then, we finally obtain the linear matrix inequality (11).

Thus, if there exist matrices $\hat{Q}_i, \hat{Y}_{0i}, \hat{Y}_{1i}, \hat{Y}_{2i}$, being a solution of the optimization problem (10) subject to (11), then (8) is a Lyapunov function for system (6), and the consensus protocol (5) with matrices calculated in accordance with (12), provides the fulfilment of stabilization (13) and consensus (2) conditions for the system (6). The theorem is proved.

Remark. The optimization problem (10) subject to (11) is a semidefinite programming problem that is solved numerically using freely distributed software packages developed based on MATLAB, for example, cvx [33] or SeDuMi [34].

NUMERICAL EXAMPLE

As an example, we consider a linear discrete-time MAS of 6 homogeneous agents, which was studied in [35] and solve the consensus control problem using the proposed approach. We deliberately consider a homogeneous MAC to reduce the amount of calculation.

During the system dynamics, the connection topology between agents switches randomly among the set of options represented by connected graphs $G_1,...,G_5$, as shown in Fig. 1. The reference set point x^* is received externally at the input of agent 1, which is a leader.

The dimensions of the agent model are $n = 2$, $m = 1$. The dynamics and control matrices are $A_i = \begin{bmatrix} 0 & 1 \\ 0{,}25 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $i = 1,...,6$. All agents are Shur stable and controllable. The initial states of the agents are chosen:

$$x_1(0) = \mathrm{col}\{50, -100\},\ x_2(0) = \mathrm{col}\{30, -60\},\ x_3(0) = \mathrm{col}\{10, -20\},$$

$$x_4(0) = \mathrm{col}\{-10, 20\},\ x_5(0) = \mathrm{col}\{-30, 60\},\ x_6(0) = \mathrm{col}\{-50, 100\}.$$

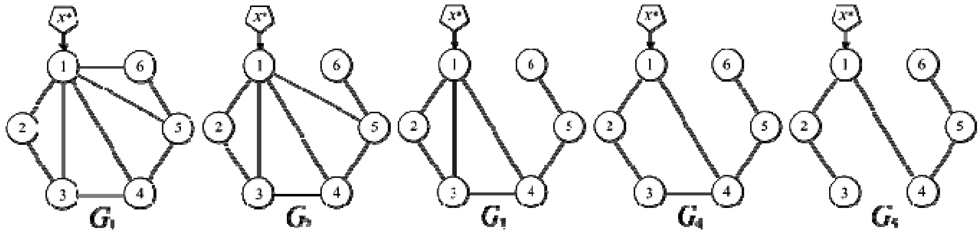


Fig. 1. The set of graphs describing the topology of connections in the MAS

We calculate the numerical solution of problem (10) subject to (11) for all versions of the MAS topology, which is presented in Fig. 1. As a result, the matrices of feedback gains, which determine the consensus protocol (5), were calculated for all agents. The analysis of the obtained results allowed to conclude that the hypothesis, put forward about the fulfilment of the statements of the proved theorem for any version of topology, described by a finite set of connected directed graphs, was experimentally confirmed, since the values of the local feedback matrices depend only on the number of neighbours with which the agent exchanges information in the current period.

For example, in a graph G_1 agents 2 and 6 are connected with two neighbours, while agents 3, 4, and 5 are connected with three neighbours. Therefore, we obtain $K_2(G_1) = K_6(G_1)$, $K_3(G_1) = K_4(G_1) = K_5(G_1)$. In a graph G_4, agents 1, 2, 3 and 5 are connected with two neighbours, and only agent 4 — with three neighbours. Accordingly, we obtain $K_1(G_4) = K_2(G_4) = K_3(G_4) = K_5(G_4) = = K_2(G_1)$, $K_4(G_4) = K_3(G_1)$. Thus, for the considered network, we obtained:

$$K_{0i} = [-0{,}324\ \ -0{,}558],\ K_{1i} = [-0{,}099\ \ -0{,}206],\ K_{2i} = [-0{,}047\ \ -0{,}057], \quad (14)$$

$$K_{0i} = [-0{,}301\ \ -0{,}480],\ K_{1i} = [-0{,}023\ \ -0{,}059],\ K_{2i} = [-0{,}031\ \ -0{,}046],$$

$$K_{0i} = [-0{,}439\ \ -0{,}762],\ K_{1i} = [-0{,}019 - 0{,}027],\ K_{2i} = [-0{,}024\ \ -0{,}041],$$

$$K_{0i} = [-0{,}634\ \ -1{,}059],\ K_{1i} = [-0{,}012\ \ -0{,}019],\ K_{2i} = [-0{,}019\ \ -0{,}037],$$

$$K_{0i} = [-0{,}752\ \ -1{,}342],\ K_{1i} = [-0{,}10\ \ -0{,}017],\ K_{2i} = [-0{,}010 - 0{,}026], \quad (15)$$

where (14) corresponds to an agent exchanging information with one neighbour in the network, and (15) — to an agent with five neighbours, respectively.

Thus, it is sufficient to solve the optimization problem (10) subject to (11) for only one agent, sequentially changing the values of the binary variables δ_{ij}, $j = 1,\dots,N,\ j \neq i$ that determine the number of neighbours with which the agent exchanges information, to calculate the values of the feedback matrices for all possible cases: from having one neighbour before having $(N-1)$ neighbours.

The plots of the changes in the values of the first and second components of the agent's state vectors with fixed topology in the absence of setting action are shown in Fig. 2. The simulation results provided that the variant of the connection topology between agents at each step was chosen randomly among the versions presented in Fig. 1, are shown in Fig. 3. Clearly in both experiments, all agents achieve a consensus, but the consensus value in the second case differs from zero, that is, there is a static error.

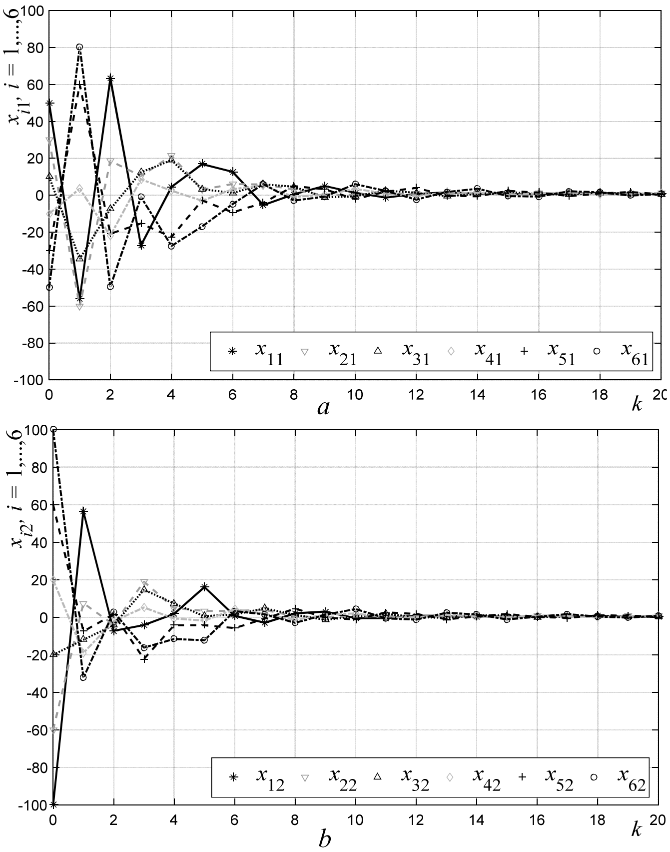


Fig. 2. Values of agents' states with fixed topology in the absence of setting action: *a* — first components of state vectors; *b* — second components of state vectors

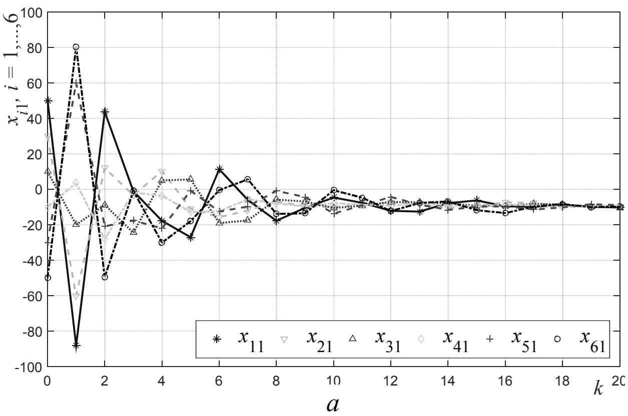
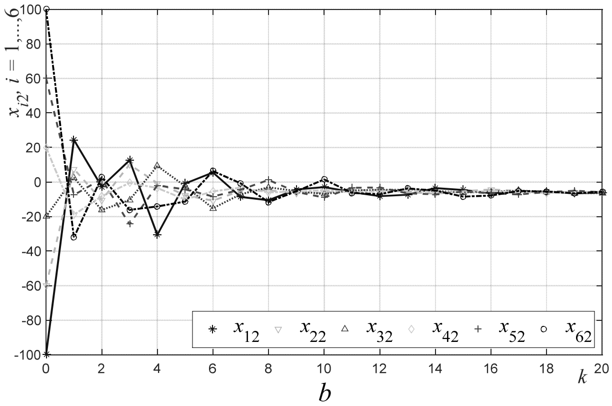


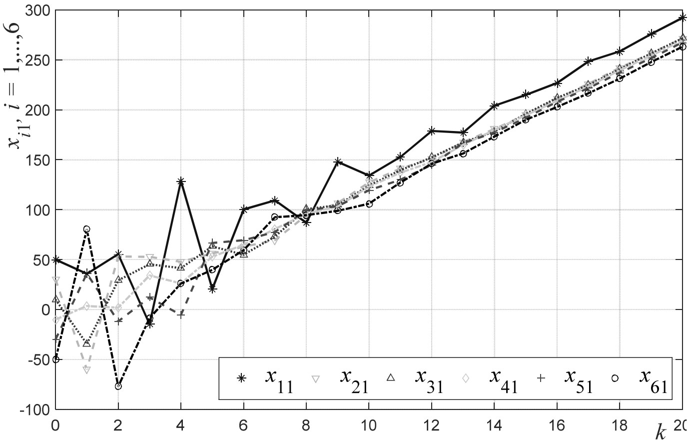
Fig. 3. Values of agents' states with switching topology in the absence of setting action: *a* — first components of state vectors; *b* — second components of state vectors



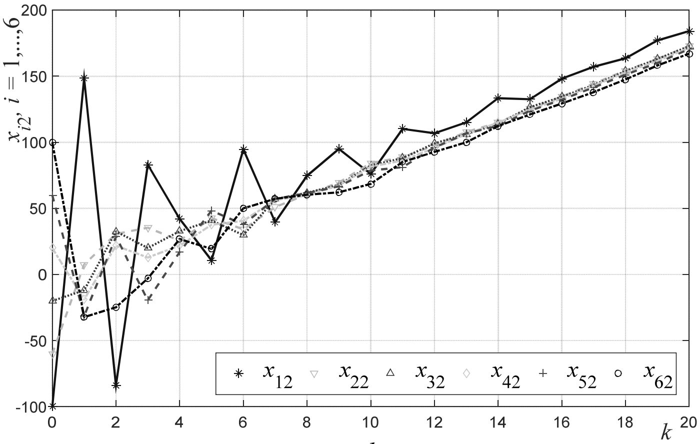
b

Fig. 3. Values of agents' states with switching topology in the absence of setting action: *a* — first components of state vectors; *b*— second components of state vectors. *End*

The simulation results of MAS with fixed topology, when the input of agent1, which is a leader, is supplied with a constant setting action $x^* = \mathrm{col}\{500, 500\}$, is shown in Fig. 4. Fig. 5 shows the results obtained for MAS with switching topology in the presence of a reference setting.



a



b

Fig. 4. Values of agents' states with fixed topology in the presence of setting action: *a* — first components of state vectors; *b* – second components of state vectors

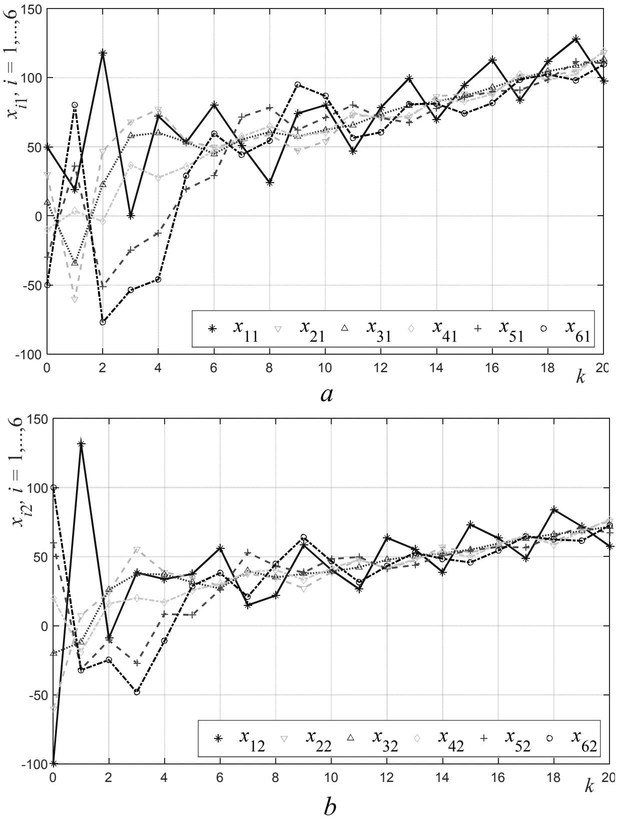


Fig. 5. Values of agent's states with switching topology in the presence of setting action: *a* — first components of state vectors; *b* — second components of state vectors

The analysis of the obtained results makes it possible to conclude that the consensus protocol (5) with the gain matrices calculated in accordance with the proposed decentralized method, can achieve consensus in a finite number of steps for any variant of the interconnection topology between agents from a given finite set.

CONCLUSIONS

The problem of decentralized consensus control of linear discrete-time multi-agent systems with switching topology in the presence of a leader is solved in this paper. A consensus protocol providing coordinating control is constructed in a feedback form with a PID controller using the deviation signal of the local agent state vector from the weighted average state vector of its neighbours. The discrete PID controller equation is presented in a recurrent form. The sufficient conditions for stabilization of the closed-loop local agent and the global consensus by constructing the quadratic Lyapunov function are obtained. Based on the invariant ellipsoid's method, the problem of local controller design is reduced to the problem of semidefinite programming, which is solved numerically. Analysis of the obtained results allowed us to conclude that the values of the gain matrices of local controllers depend on the number of neighbours with which the agent exchanges information in the current period. The significance of this study is to develop a practically realizable method for solving the consensus control problem of discrete-time multi-agent systems with switching topology based on a decentralized approach, which does not require using the graph Laplacian that describes the connection topology between agents. The proposed approach may be further expanded to solve the consensus control problems of multi-agent systems in the presence of delays in measurements or in the process of information exchange between agents.

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СИНТЕЗ ДЕЦЕНТРАЛІЗОВАНОГО КОНСЕНСУСНОГО КЕРУВАННЯ ДЛЯ МУЛЬТИАГЕНТНИХ ДИСКРЕТНИХ СИСТЕМ З КОМУТАЦІЙНОЮ ТОПОЛОГІЄЮ ЗА НАЯВНОСТІ ЛІДЕРА / Ю.І. Дорофєєв, Л.М. Любчик, М.М. Малько

Анотація. Розглянуто задачу консенсусного керування лінійними мультиагентними дискретними системами (МАС) з комутаційною топологією за наявності лідера. Мета консенсусного керування полягає у зведенні станів усіх агентів до стану лідера з одночасним забезпеченням стійкості локальних агентів, а також МАС у цілому. На відміну від традиційного підходу, який використовує концепцію розширеної динамічної моделі мультиагентної системи та лапласіан графу комунікаційної топології, запропоновано підхід на основі декомпозиції, який передбачає незалежне проєктування локальних регуляторів. Закон керування вибирається у вигляді розподіленого зворотного зв’язку з дискретними ПІД-регуляторами. Задачу синтезу локальних регуляторів за допомогою методу інваріантних еліпсоїдів зведено до набору задач напіввизначеного програмування. Достатні умови стабілізації агентів та досягнення глобального консенсусу отримано за допомогою техніки лінійних матричних нерівностей. Наявність інформації про кінцевий набір можливих конфігурацій зв’язків між агентами дозволяє синтезувати локальні регулятори в автономному режимі на етапі проєктування. Ефективність запропонованого підходу продемонстровано за допомогою числового прикладу.

Ключові слова: мультиагентна система, консенсусне керування, комутаційна топологія, ПІД регулятор, метод інваріантних еліпсоїдів, лінійна матрична нерівність, задача напіввизначеного програмування.