UDC 004.942
DOI: 10.20535/SRIT.2308-8893.2025.3.10

MATHEMATICAL MODELING OF INFORMATION DIFFUSION PROCESS BASED ON THE PRINCIPLES OF THERMAL CONDUCTIVITY

V.O. RETS, E.V. IVOHIN

Abstract. Information diffusion, a fundamental process underlying societal evolution and decision-making, shares intriguing analogies with thermodynamics. This paper presents a mathematical model that bridges these domains by proposing an analogy between thermodynamics and information theory. The study introduces a solved heat equation as a foundational framework to model information diffusion within societal contexts. The specified societal conditions embedded within the solved heat equation are central to this model. These conditions encapsulate the susceptibility of a society to assimilate new information, the constraints dictating the number and nature of available information sources, and the dynamics of information distribution characterized by its aggressiveness. The relationship between information diffusion and thermodynamics lies in their inherent propensity to seek equilibrium or optimal states. Leveraging this analogy, the solved heat equation becomes a potent tool to simulate the dynamics of information spread, analogous to the flow of thermal energy within physical systems. This work aims to stimulate further inquiry into the parallels between thermodynamics and information theory, presenting a theoretical framework and software implementation that open new avenues for understanding and modeling information diffusion dynamics within complex societal systems.

Keywords: information diffusion, heat equation modeling, partial differential equations, information propagation, temperature distribution, physical process analogy, information flow analysis, system analysis, mathematical modeling.

INTRODUCTION

In today's information world, the distribution of information is a key process that affects the evolution of society, decision-making paradigms, and technological progress. The goal of information is to empower and educate consumers and help them make choices that can affect everything from choosing a book to choosing a government. However, information varies in both quality and impact. In recent years, the number of accidental and intentional consequences of using social network platforms to spread misinformation has increased [1]. If we perceive news as a description of current, real and important events [2] that affect people [3], then fake news is a whole ecosystem of information that includes both the distribution of true information and the creation and distribution of misinformation [3, 4]. Fake news is used to create disinformation articles, hoaxes, rumors, parodies, incorrect editorials, incorrect facts, etc. Such a variety of goals, channels, sources and motivations makes it difficult to understand their spread.

In order to learn how to properly use information as a tool that would play in favor of its user, you need to be able to model it. To do this, you can present a mathematical model of information distribution based on predetermined parameters, such as:

- perception of new information by society;
- imitations on the number and "aggressiveness" of information sources;
- time limit during which the news distribution process takes place.

The diffusion approach can be singled out among the most well-known and meaningful approaches used in the research of information distribution processes [5, 6]. Using the results of research on the relationship between the equations of thermodynamics and information theory, it is possible to draw a parallel and build a diagram of heat distribution in a one-dimensional rod to model the distribution of information in a linear graph of social relations, considering the linear graph as a subgraph of the tree of the social hierarchy [7].

HEAT EQUATION

The heat conduction equation is a well-known fundamental differential equation in partial derivatives, which is widely used in physics, engineering and various scientific fields. It describes how the distribution of heat changes over time in a certain region of space. Mathematically, it relates the rate of temperature change to the spatial distribution of temperature and time.

The general form of the heat conduction equation in one-dimensional space is represented by the formula [8]:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

where the function $u(x,t)$ represents the temperature or distribution of heat in the material at a certain place and at a certain time, t denotes a time variable, x denotes a spatial coordinate, α is the coefficient of thermal conductivity, which is a constant for each material that characterizes the rate of diffusion heat through the material.

FORMULATION OF THE PROBLEM

In the real world, it is appropriate to consider the process of information distribution with several sources that have certain limitations. These can be television channels, Internet blogs, or even just people who in one way or another notify people around them about news or facts known to them.

Having set the goal of raising public awareness of a certain news (which can be both true and deliberately incorrect), it is necessary to calculate the optimal location of information sources to achieve the desired level of awareness – based on pre-established constraints, among which:

- the maximum number of sources;
- "aggressiveness" of the source, i.e., how quickly the parts of society connected with it will receive new information [9];
- maximum allotted time.

In this case, knowing the data on the attitude and receptivity of the society to new information, it is possible to draw a parallel with the processes characteristic to that of thermodynamics [10] — the distribution of information in society can be formalized on the basis of the propagation of heat in the material. At the same time, its thermal diffusivity can be considered as susceptibility to new information, and the "aggressiveness" of the sources will be determined by the maximum temperature of the heating element. For simplicity of initial calculations, it can be assumed that information is not lost in the system, that is, there is no "forgetting" effect, which means no heat exchange of the material with the surrounding environment. To conduct the research, we will consider a linear graph of social relations, the mechanistic analogue of which will be a completely isolated one-dimensional rod with a thermal conductivity coefficient α. Then the achievement of the desired level of awareness can be considered as heating the rod at least by Δt degrees with the help of heating sources of temperature T_1 evenly located in the rod for the maximum time $t_{\max}$ with the initial temperature of the rod equal to zero, which is in turn equal to the temperature of the environment.

PROPOSITION

Given the given restrictions (on the number of heating elements, their maximum temperature, and heating time), we can conclude that under certain conditions it will not be possible to heat the entire rod with one source, so it is worth considering splitting the rod into equal parts, in the center of each of which a heating element can be placed. Thus, by dividing the general problem of calculating the temperature distribution $U_L(x,t)$ along a rod of length L into equal subtasks of heating parts of the rod $U_i(x,t)$, $i=\overline{1,n}$, where n is the number of subtasks, we can study only the case of heating one of them to obtain a complete picture of the experiment:

$$U_L(x,t)=\begin{cases} U_1(x,t), & x\in[x_0,x_1]; \\ U_2(x,t), & x\in[x_1,x_2]; \\ \dots\dots\dots & \dots\dots\dots \\ U_n(x,t), & x\in[x_{n-1},x_n], \end{cases} \quad \forall i\in\overline{1,n} \;\; x_i-x_{i-1}=\frac{L}{n}.$$

Since the heating of a part of the rod occurs in the middle equally in both directions, the subtask is decomposed into two more identical problems of calculating $U_{i/2}(x,t)$ with the heating of a part of the rod on one side, $i=\overline{1,n}$. At the same time, for each $i=\overline{2,n}$, the boundaries of the neighboring problems U_{i-1} and U_i will not have heat exchange due to the same heat dynamics (due to the same size of the neighboring subtasks' definition regions and the same power of their heating elements), which means that in the $U_{i/2}$ problems, we can consider a completely isolated rod except for the heated side. Due to the completely identical conditions for each part of the rod, the $U_{i/2}(x,t)$ problems are identical, so their solution can be denoted as $u(x,t)$. The tasks will be as follows: find the solution to the equation

$$\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}$$

with boundary conditions

$$u(0,t) = T_1, \quad \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} = u_x(L,t) = 0,$$

and the initial condition $u(x,0) = 0$.

SOLUTION OF THE SUBTASK

First, let's determine the steady solution u_s, which determines the equilibrium state. By solving the differential equation

$$\frac{d^2 u_s}{dx^2} = 0$$

we have

$$u_s = Ax + Bu(0) = B = T_1 u'(L) = A = 0 u_s = T_1 .$$

In this case, until the moment of equilibrium, $u(x,t)$ will be influenced by the function $\nu(x,t)$, then $u(x,t)$ can be written as:

$$u(x,t) = T_1 + \nu(x,t) . \qquad (1)$$

We solve a similar problem with respect to ν:

$$\frac{\partial \nu}{\partial t} = \alpha \frac{\partial^2 \nu}{\partial x^2} ,$$

with boundary conditions

$$\begin{cases} \nu(x,t); \\ \left.\dfrac{\partial \nu(x,t)}{\partial x}\right|_{x=L} = \nu_x(L,t) = 0, \end{cases}$$

and initial condition

$$\nu(x,0) = -T_1.$$

Apply the method of separating variables

$$\nu(x,t) = X(x)T(t) . \qquad (2)$$

Substitute into the equation and get

$$X(x)T'(t) = \alpha X''(x)T(t) , \quad \frac{X''}{X} = \frac{T'}{\alpha T} = -\lambda .$$

We have two equations:

$$X'' + \lambda X = 0 , \qquad (3)$$

$$T' + \alpha\lambda T = 0. \qquad (4)$$

Solve (3):

$$\begin{cases} X'' + \alpha X = 0, \\ X(0) = 0, \\ X'(L) = 0. \end{cases}$$

Considering the following intervals

$$1.\ \lambda = \beta^2 > 0 ,\ 2.\ \lambda = 0 ,\ 3.\ \lambda = -\gamma^2 < 0 ,$$

we obtain that the solutions are nontrivial only for the first interval, and we have the following values:

$$\lambda_n = \frac{\pi\left(n+\frac{1}{2}\right)}{L}, \quad n = 0,1,2,\dots\ .$$

Therefore, the solutions of (3) take the following form $X_n(x) =$ $= D_n \sin\frac{\pi\left(n+\frac{1}{2}\right)}{L}x$, $n = 0,1,2,\dots$.

Let's return to equation (4) $T' + \alpha\lambda T = 0$. The solution to this equation is as follows $T(t) = Ae^{-\alpha\lambda t}$.

For each $X_n(x)$ with an eigenvalue λ_n, we set $T_n(t)$ to $T_n(t) = A_n e^{-\alpha\lambda_n t}$.

Now we can write down the general form of the solution to (2):

$$\nu(x,t) = \sum_{n=0}^{\infty} A_n \sin\frac{\pi\left(n+\frac{1}{2}\right)x}{L} e^{-\alpha\lambda_n t}.$$

Consider the initial condition for finding A_n:

$$\varphi(x) = \sum_{n=0}^{\infty} A_n \sin\frac{\pi\left(n+\frac{1}{2}\right)x}{L} = -T_1.$$

Using the orthogonality property of the eigenfunctions of the Sturm–Liouville problem [11]:

$$A_n = \frac{X_n, \varphi}{X_n, X_n}$$

from where

$$A_n = -\frac{2T_1}{L}\int_0^L \sin\frac{\pi\left(n+\frac{1}{2}\right)x}{L}dx,$$

$$A_n = -\frac{2T_1}{L}\left(-\frac{2L\cos\frac{2\pi n+\pi}{2} - 1}{\pi(2n+1)}\right) = \frac{4T_1\left(\cos\frac{2\pi n+\pi}{2} - 1\right)}{\pi(2n+1)}.$$

Substituting this value into (1), we have a solution to the problem for half of one part of the rod:

$$u(x,t) = \frac{4T_1}{\pi}\sum_{n=0}^{\infty}\frac{\cos\frac{2\pi n+\pi}{2} - 1}{2n+1}\sin\frac{\pi\left(n+\frac{1}{2}\right)x}{L} e^{-\alpha\frac{\pi\left(n+\frac{1}{2}\right)}{L}t} + T_1$$

APPLICATION OF THE SOLUTION

Substituting $t = t_{\max}$ into the resulting formula, we can obtain the function $u(x)$ of the rod temperature distribution at the maximum permissible time point. Since in the context of the task at hand, it is necessary to raise the temperature of the rod by at least Δt, i.e., from 0 to a certain $T_2 < T_1$, it is necessary to find such $x = x_{T_2}$, at which T_2 will be achieved. To do this, we can use the dichotomy method, which will give us half the lengths of the maximum size parts of the rod. After that, the partition length L_n can be found by the formula:

$$L_p = \frac{L}{2x_{T_2}}.$$

Further, we can perform additional optimization procedures using similar approaches:

- reduce the heating time so that the temperature at the point x_{T_2} does not rise above T_2.
- reduce the temperature of the heating element so that the temperature at the point x_{T_2} does not rise above T_2 during the specified time.

IMPLEMENTATION

In the course of the study, the described algorithm was implemented in Python with visualization using the methods of the Matplotlib package and two optimization options — time and maximum temperature.

As an example, let's take the problem of heating an aluminum rod with a length of 30 centimeters, a maximum temperature of 100º, and a maximum time of 2 minutes. After performing the described calculations, we get that in order to achieve the desired result, it is necessary to break the rod into 6 equal parts. The steps of simulating the heating process using the described software can be seen in Figs. 1–3.

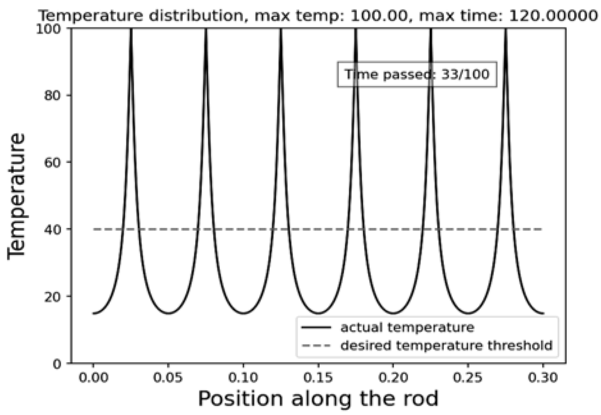

Fig. 1. The temperature distribution at the beginning of the simulation

Fig. 1 shows the temperature distribution at the beginning of the simulation, where heating took place for a third of the maximum time. It can be noted here that the distribution corresponds to the described model — at the heating points, the temperature is maximum from the beginning of the simulation, and in the neighboring regions it is distributed according to the distance from the heating elements. Also, the desired temperature along the rod is indicated in gray in the figures — it should be achieved within a given time.

In Fig. 2 shows the distribution at the time of two-thirds of the maximum time — the temperature along the entire rod has increased significantly, steadily approaching the desired level.

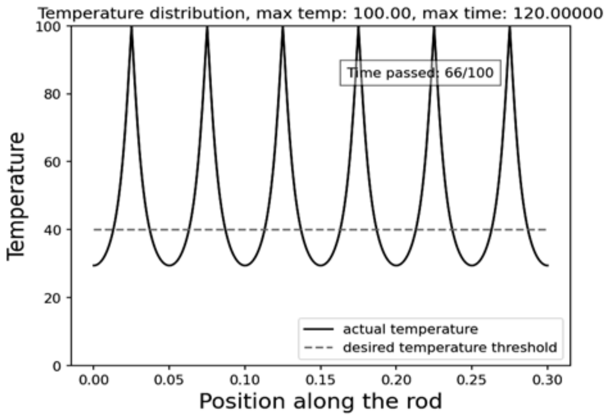

Fig. 2. The distribution at the time of two-thirds of the maximum time

Finally, Fig. 3 shows the final state — the desired temperature has been reached along the entire rod, which means that the program has completed execution.

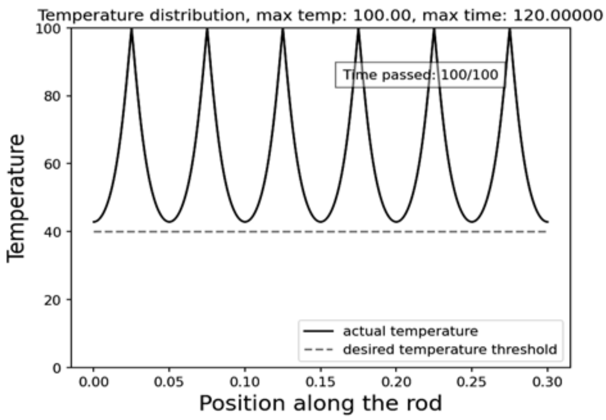

Fig. 3. The desired temperature has been reached along the entire rod

It should be noted that the temperature scale can be either in Kelvin or in Celsius, since the heating process in the context of the task depends only on the temperature difference between the heating source and the initial state, and the calculations are performed in the Celsius–Kelvin dimensional scale.

The proposed approach made it possible to study and compares the results obtained in the process of information distribution in social groups. Taking into account the assumed analogy between the process of heat exchange and information distribution, a comparison was made using the indicators of iron as a material whose thermal conductivity determines the part of society with a low level of perception of new information, and aluminum to describe the representatives of society who are prone to easy assimilation and perception of information. The length of the respective material was used to represent the conditional size of the studied society: 3 meters for small groups and 50 meters for large groups. The threshold value of heating corresponding to the level of awareness was set at 40º, and the heating source for the level of exposure at 100º. The maximum time is 20 minutes (see results in Table).

Table

Sources / Perception type	The required number of sources	Conventional length of a separate zone of influence
High perception, small society	6	0.5
Low perception, small society	24	0.12
High perception, big society	93	0.54
Small perception, big society	389	0.13

CONCLUSION

This article investigates one of the approaches to the mechanistic application of models based on the use of the heat conduction equation to model and simulate the process of information multi-source distribution in a society. The problem is solved by analogizing a linear graph of social relations as a one-dimensional homogeneous rod, which makes it possible to find the required number of information sources to achieve the required level of selected information awareness in society within a given time. To conduct a simulation study of the found model, a software tool was created in Python and the Matplotlib visualization package, which finds the required number of sources under the specified conditions and conducts a simple visualization of the temperature distribution during the process of information distribution ("heating"). A qualitative improvement of the results obtained could be to perform similar calculations in two-dimensional space, which would allow for a more accurate modeling of connections in social groups. An important result should also be the study of the heating processes of a non-insulated rod or plate, for which the factor of information forgetting is added. This approach can be used to model real information diffusion processes to analyze existing news distribution, or to develop and simulate information campaigns for the effective distribution of desired information, such as advertising.

REFERENCES

1. "Digital Wildfires," *World Economic Forum*, 2018. Available: http://reports. weforum.org/global-risks-2018/digital-wildfires/
2. James W. Kershner, *The Elements of News Writing*. Boston, MA: Pearson Allyn and Bacon, 2005.
3. Brian Richardson, *The Process of Writing News: From Information to Story*. Boston, MA: Pearson, 2007.
4. Claire Wardle, "Fake News. It's Complicated," *Medium*. Available: https://medium. com/1st-draft/fake-newsits-complicated-d0f773766c79
5. E.V. Ivohin, O.F. Voloshyn, M.F. Makhno, "Modeling of Information Dissemination Processes Based on Diffusion Equations with Fuzzy Time Accounting," *Cybernetics and Systems Analysis*, 57(6), pp. 896–905, 2021. doi: https://doi.org/10.1007/ s10559-021-00416-z
6. E.V. Ivokhin, L.T. Adzhubey, Y.A. Naumenko, M.F. Makhno, "On one approach to using of fractional analysis for hybrid modeling of information distribution proc-

esses," *System Research and Information Technologies*, no. 4, pp. 128–137, 2021. doi: https://doi.org/10.20535/SRIT.2308-8893.2021.4.10
7. M. Boguná, R. Pastor-Satorras, A. Díaz-Guilera, A. Arenas, "Models of social networks based on social distance attachment," *Physical review E*, 70(5), 056122, 2004. doi: 10.1103/PhysRevE.70.056122
8. John Rozier Cannon, "The one-dimensional heat equation," *Encyclopedia of Mathematics and its Applications (23)*. Cambridge University Press, 1984. doi: https://doi.org/10.1017/CBO9781139086967
9. M. MacKuen, "Exposure to information, belief integration, and individual responsiveness to agenda change," *American Political Science Review*, 78(2), pp. 372–391, 1984. doi: https://doi.org/10.2307/1963370
10. S. Diakonova, S. Artyshchenko, D. Sysoeva, I. Surovtsev, M. Karpovich, "On the application of the thermal conductivity equation to describe the diffusion process," in *E3S Web of Conferences*, vol. 175, p. 05050. EDP Sciences, 2020. doi: https://doi.org/10.1051/e3sconf/202017505050
11. M.A. Al-Gwaiz, *Sturm-Liouville theory and its applications*. Springer, 2008, 264 p. doi: https://doi.org/10.1007/978-1-84628-972-9

Received 15.03.2024

INFORMATION ON THE ARTICLE

Eugene V. Ivokhin, ORCID: 0000-0002-5826-7408, Taras Shevchenko National University of Kyiv, Ukraine, e-mail: ivohin@univ.kiev.ua

Vadym O. Rets, ORCID: 0009-0007-8632-7010, Taras Shevchenko National University of Kyiv, Ukraine, e-mail: vadym.rets@gmail.com

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСУ ПОШИРЕННЯ ІНФОРМАЦІЇ НА ОСНОВІ ПРИНЦИПІВ ТЕПЛОПРОВІДНОСТІ / В.О. Рець, Е.В. Івохін

Анотація. Поширення інформації являє собою фундаментальний процес, що лежить в основі суспільної еволюції та прийняття рішень і має доведені аналогії з термодинамікою. Запропоновано математичну модель, що дозволяє продемонструвати можливість застосування законів та принципів явища теплопровідності для моделювання процесів поширення інформації. Розглянуто оригінальну модель на основі скалярного рівняння у частинних похідних для формалізації процесів поширення інформації в суспільному контексті. Визначено характерні умови, які знайшли відображення у розв'язку розглянутого рівняння теплопровідності; відзначено сприйнятливість суспільства до засвоєння нової інформації, обмеження, що визначаються кількістю та властивостями доступних джерел інформації, а також динаміку поширення інформації, яка відзначається своєю агресивністю. Взаємозв'язок між поширенням інформації та термодинамікою полягає у властивій їм схильності шукати рівноважний або оптимальний стан. Ураховуючи цю подібність, розв'язане рівняння теплової енергії можна використовувати як потужний інструмент для моделювання динаміки поширення інформації, аналогічно потоку теплової енергії у фізичних системах. Ця робота має на меті стимулювати подальше дослідження паралелей між термодинамікою та теорією інформації, подаючи теоретичну основу та програмну реалізацію, що визначають нові шляхи для розуміння та моделювання динаміки поширення інформації в складних суспільних системах на основі використання різних моделей фізичних процесів.

Ключові слова: дифузія інформації, моделювання рівнянь теплопровідності, диференціальні рівняння в частинних похідних, поширення інформації, розподіл температури, аналогія фізичного процесу, аналіз інформаційних потоків, системний аналіз, математичне моделювання.