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INTELLIGENT OPTIMAL CONTROL OF NONLINEAR DIABETIC POPULATION DYNAMICS SYSTEM USING A GENETIC ALGORITHM

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Abstract. Diabetes is a chronic disease affecting millions of people worldwide. Several studies have been carried out to control the diabetes problem, involving both linear and non-linear models. However, the complexity of linear models makes it impossible to describe the diabetic population dynamic in depth. To capture more detail about this dynamic, non-linear terms were introduced into the mathematical models, resulting in more complicated models strongly consistent with reality (capable of re-producing observable data). The most commonly used methods for control estimation are Pantryagain's maximum principle and Gumel's numerical method. However, these methods lead to a costly strategy regarding material and human resources; in addition, diabetologists cannot use the formulas implemented by the proposed controls. In this paper, the authors propose a straightforward and well-performing strategy based on non-linear models and genetic algorithms (GA) that consists of three steps: 1) discretization of the considered non-linear model using classical numerical methods (trapezoidal rule and Euler–Cauchy algorithm); 2) estimation of the optimal control, in several points, based on GA with appropriate fitness function and suitable genetic operators (mutation, crossover, and selection); 3) construction of the optimal control using an interpolation model (splines). The results show that the use of the GA for non-linear models was successfully solved, resulting in a control approach that shows a significant decrease in the number of diabetes cases and diabetics with complications. Remarkably, this result is achieved using less than 70% of available resources.

Keywords: optimal control, differential equation, diabetes, genetic algorithms, artificial intelligence, intelligent local search.

INTRODUCTION

Diabetes is a major public health problem. Diabetes is a major public health problem and one of the most dangerous common diseases and is characterized by high blood sugar [2]. Diabetes is the root of many diseases and costs many lives. It is a serious chronic disease that occurs when the body does not properly use the insulin it produces or when the pancreas does not produce enough insulin. There are three types of diabetes: Type 1 diabetes, Type 2 diabetes and Gestational Diabetes Mellitus (GDM) [3].

The number of people with diabetes has increased exponentially in recent times. According to the World Health Organization (WHO) and the Interna- tional Diabetes Federation (IDF) [1; 4], 463 million people had diabetes in 2019 and this number is expected to reach 578 million in 2030 and 700 million in 2045 [11; 9].

The mathematical modeling of the phenomenon of diabetes is the subject of several researchers in different mathematical fields. These include ordinary differential equations (ODE) that study, for example, the diabetic population as found in [5–8], there are also research works that study the phenomenon of diabetes by modeling has partial differential equations (PDE). Likewise, studies on this phenomenon are carried out using delay differential equations (DDE), and others, for example stochastic differential equations (SDE) and integro-differential equations (IDE), Fredholm integral equations (FIE) [15–18].

In [18] the authors have given a thorough review of the delay differential equation models and they are presented with some computational results and brief summaries of the theoretical results for the cases of the insulin ultradian oscillation models and the models for the diagnostic tests.

The modeling of natural phenomena is a very successive task to deal with any phenomenon [40]. In the first modeling of the phenomenon of diabetes, two types of population were considered, pre-diabetic and diabetic. The authors of [5] modeled this natural phenomenon as a system of linear ordinary differential equations of the first degree. Then, in 2007 [10], a modification was made to the above system. This time, another type of population was considered, namely diabetic patients with complications.

Several studies were performed for this model, they show that the system is well defined, also the stability and determination of equilibrium points was done. Then in the year 2014 [6], the authors is thought to try to reduce the neg- ative effect of the phenomenon of diabetes, for this they proposed an approach of optimal control that help to minimize as much as possible the spread of the phenomenon, focusing on the study of the diabetic population. Three types of diabetic patients are considered, patients who become diabetic for different reasons, which may be genetic or related to a negative lifestyle. The other two types are diabetic patients without complications and diabetic patients with complications. The modeling of the mathematical model is well explained in [6], and the existence and uniqueness of solution is well shown, also the existence of control. In the ten years, the studies carried out with this control have shown their effectiveness in reducing the number of diabetic patients, which shows the success of this strategy of searching for an optimal control to control this phenomenon. A dynamic 6-compartment control system was proposed for the study of the diabetes phenomenon in [11], they divided the population in general as follows: healthy people H, pre-diabetic patients due to genetics which is denoted by P, pre-diabetic patients due to lifestyle denoted by E. The other three types are di- abetic patients without complications and diabetic patients with complications, which are denoted by D and C respectively. In [11], they paid attention to the fact that there are several types of consciousness that are directly related to diabetes, there are genetic influences and bad lifestyle, on the other hand there is the psychology of the person. For this reason, they proposed optimal controls that take these influences into account.

In [12], another reformulation of dynamic model of diabetic population was proposed. They resulted that the factors most related to diabetes or to the fact that a person becomes diabetic are the genetic factors and a bad lifestyle. To protect diabetic patients, an awareness program was proposed using media and education; psychological follow-up was also considered and medical treatment, this time they grouped patients according to their age through a continuous dynamic system. They divided the general population into 4 different compartments given in the following form: pre-diabetic patients due to genetics, pre-diabetic patients due to lifestyle, diabetic patients with complication and without complication.

In this work we propose a very sample and performance strategy based on non-linear models and genetic algorithm (GA)s that processes into three steps: 1) discretization of the considered non-linear model using classical numerical methods (trapezoidal rule and Euler–Cauchy algorithm); 2) estimation of the optimal control, in several points, based on GA with appropriate fitness function and suitable genetic operators (mutation, crossover, and selection); 3) construction of the optimal control using interpolation model.

This paper is organized as follows: The second section provides an introduction on the importance of using genetic algorithms to solve optimal control problems. The third section presents two dynamic control systems, one is the kernel linear system and the other is the nonlinear system proposed in recent years, we have given the theoretical summary that has been done on both models. The fourth section presents the minimization of the objective functions of each control system of the diabetic population by the genetic algorithm and we compare the results found by the results of the classical method. Finally, we end the paper with a general conclusion.

GENETIC ALGORITHM

In [9] Boutayeb et al., in recent years artificial intelligence is invading all fields, it has also become an essential part of human life. For example, in this work, we will use the artificial intelligence methods to treat the optimal control problems associated with a diabetic population [29; 14].

In this article, we deal with a very complicated phenomenon, because several effects come into play, for example genetic influence, age, lifestyle and others.

Genetic algorithms are artificial intelligence methods and are heuristic search techniques that are very simple to handle [30; 31]. Genetic algorithms (GAs) are search optimization algorithms based on three essential operators, selection, crossover and mutation [21]. In general, GAs were first developed by Holland and are derived from Darwin's theory of evolution [20]. In the first step, a population of initial solutions, called chromosomes, is randomly selected, then these solutions are evaluated by the objective function or fitness function, if the solution has a very high performance then this is the solution we are looking for, otherwise, we move to the crossover and mutation step which generates a new solution from the initial solution, likewise this solution generated and evaluated by the objective function, and so on until we find the best solution. Genetic algorithms are very effective in complex optimization problems more than simple methods, which sometimes fail to achieve a certain problem due to its complexity [32].

They are used in many research areas and are very useful in real world applications, because they are very simple and give the best solutions. Classical optimization methods, which are purely computational methods, start with a single initial solution and then search for the optimal solution, but the genetic algorithm starts with an initial population of candidates and then searches for the best optimal solution in the search space [28].

Optimal control problems are also among the optimization problems that have been solved by the GA. The real beginning of the use of GA for dynamic control systems is in the years 1992 by Krishnakumar and Goldberg [22] who gave a start to GA in a very important discipline of applied mathematics [23; 26; 27]. The main objective of our work is to achieve a significant reduction in the number of diabetics without complications and with treatable complications, which involves the search for optimal control to achieve this goal. The dynamic PEDC model is a non-linear mathematical model representing the evolution of the diabetic population and the influence of uncomplicated diabetic patients on pre-diabetics. The advantage of using the GA to solve this type of optimal control problem is that we do not need to determine the adjoint values or characterize the control in any specific way. We only need to treat the problem well and specify the objective function we need to minimize and its various constraints.

THE MODELS

In this section, we will focus on different dynamic and controlled systems for diabetic patients found in the literature that minimize the negative impact of this phenomenon. First, we will start with the dynamically controlled Kernel system, which allows the study of diabetic patients without entering their bodies.

The EDC model

In [13] Boutayeb et al. have proposed an optimal control approach modeling the progression from pre-diabetes to diabetes with and without complications. Three types of diabetic patients are considered, patients who become diabetic for different reasons, which may be genetic or related to a negative lifestyle, this type is noted by *E*. The other two types are diabetic patients without complications and diabetic patients with complications, which are denoted by *D* and *C* respectively. We conclude that the diabetic population *N* is divided into three types such that $N = N(t) = E(t) + C(t) + D(t)$ at time t:

$$\begin{cases} \dfrac{dE(t)}{dt} = I - (\mu + \beta_3 + \beta_1)E(t); \\ \dfrac{dD(t)}{dt} = \beta_1 E(t) - (\mu + \beta_2)D(t) + \gamma C(t); \\ \dfrac{dC(t)}{dt} = \beta_3 E(t) + \beta_2 D(t) - (\mu + \gamma + \nu + \delta)C(t). \end{cases}$$

The modeling of the model is explained in [13], the model contains eight parameters: $I, \mu, \beta_1, \beta_2, \beta_3, \gamma, \nu$ and δ, which were estimated in [10]. The control of this phenomenon is the subject of several research works, due to the fact that this phenomenon has social burdens that should be reduced. For this purpose the authors proposed to control the transition rate from diabetes with complications to diabetes without complications.

The controlled model is given by the following system:

$$\begin{cases} \dfrac{dE(t)}{dt} = I - (\mu + (\beta_3 + \beta_1)(1 - u(t)))E(t); \\ \dfrac{dD(t)}{dt} = \beta_1(1 - u(t))E(t) - (\mu + \beta_2(1 - u(t)))D(t) + \gamma C(t); \\ \dfrac{dC(t)}{dt} = \beta_3(1 - u(t))E(t) + \beta_2(1 - u(t))D(t) - (\mu + \gamma + \nu + \delta)C(t), \end{cases} \quad (1)$$

where u is a control. The objective function we are trying to minimize is given by:

$$J(u) = \int_0^T (D(t) + C(t) + Au^2(t))dt,$$

where A is a positive weight that balances the size of the terms. U is the control set defined by:

$$U = \{u / \text{uismeasurable}, \ 0 \le u(t) \le 1, \ t \in [0, T]\}.$$

The goal is to find a control u^* in U that minimizes the objective function:

$$J(u^*) = \min_{u \in U} J(u).$$

The system (1) is well defined, moreover theorems 3.2, 3.3, 3.4 and 3.5 in [13] have shown respectively the existence and uniqueness and the positivity of the solution, the existence of an optimal control and the characterization of this control for this system (1).

Characterisation of the control optimal. The principle of maximum Pontryagin and Hamiltonian has been used to characterize the optimal control, and the adjoint variables. After mathematical calculations and demonstrations that can be found in [13], we find that the adjoint variables are defined by:

$$\begin{cases} \lambda_1' = (\lambda_1 - \lambda_2)(1-u^*)\beta_1 + (\lambda_1 - \lambda_3)\beta_3 + \mu\lambda_1; \\ \lambda_2' = -1 + (\lambda_2 - \lambda_3)(1-u^*)\beta_2 + \mu\lambda_2; \\ \lambda_3' = -1 + (\lambda_3 - \lambda_2)\gamma + \lambda_3(\mu + \nu + \delta) \end{cases}$$

or λ_1, λ_2 and λ_3 are the adjoint variables With conditions: $\lambda_1(T)=\lambda_2(T)=$ $=\lambda_3(T)=0$ and E^*, D^* and C^* are the solutions of system (1) and the control is defined as :

$$u^* = \min\left(1, \max\left(0, \frac{1}{2A}[E^*\beta_1(\lambda_2 - \lambda_1) + E^*\beta_3(\lambda_3 - \lambda_1) + D^*\beta_2(\lambda_3 - \lambda_2)]\right)\right).$$

This study, which was conducted over a period of ten years, summarizes that the optimal control approach is very effective in reducing the effect of this phenomenon, which is demonstrated in the experimental results found.

The $HP\ EDC_S C_T$ model

An improvement of the EDC model was proposed in 2020 by Kouidere et al. [11] noted $HPEDC_T C_S$. This time by a nonlinear controlled dynamic system. Six types of populations are considered, people who are healthy H, pre-diabetic patients due to genetics which is denoted by P , pre-diabetic patients due to lifestyle denoted by E. The other three types are diabetic patients without complications and diabetic patients with complications, which are denoted by D and C respectively. We conclude that the diabetic population N is divided into six types such that $N = N(t) = H(t) + P(t) + E(t) + D(t) + C_S(t) + C_T(t)$ at time t:

$$\begin{cases} \dfrac{dH(t)}{dt} = I - (\mu + \theta_1 + \theta_2)H(t); \\ \dfrac{dP(t)}{dt} = \theta_1 H(t) - (\mu + \beta_1 + \beta_3)P(t); \\ \dfrac{dE(t)}{dt} = \theta_2 H(t) - (\mu + \gamma)E(t); \\ \dfrac{dD(t)}{dt} = \beta_1 P(t) + \gamma E(t) - \alpha_1 \dfrac{D(t)E(t)}{N} - (\mu + \beta_2 + \eta_2)D(t); \\ \dfrac{dC_T(t)}{dt} = \beta_3 P(t) + \beta_2 D(t) + \alpha_1 \dfrac{D(t)E(t)}{N} - \alpha_2 \dfrac{C_T(t)E(t)}{N} - (\mu + \eta_1)C_T(t); \\ \dfrac{dC_S(t)}{dt} = \eta_2 D(t) + \eta_1 C_T(t) + \alpha_2 \dfrac{C_T(t)E(t)}{N} - (\mu + \delta_1)C_S(t); \end{cases} \quad (2)$$

with $H(0) \ge 0$, $P(0) \ge 0$, $E(0) \ge 0$, $D(0) \ge 0$, $C_T(0) \ge 0$, and $C_S(0) \ge 0$.

The modeling of the model is explained in [11], the model contains twelve parameters: $I, \mu, \theta_1, \theta_2, \eta_1, \eta_2, ,\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma, \nu$ and δ which were estimated in [11]. The objective remains to control the transition rate from prediabetes to diabetes with complications and transition from diabetes with complications to diabetes without complications.

The controlled model is given by the following system:

$$\begin{cases} \dfrac{dH(t)}{dt} = I - (\mu + \theta_1 + \theta_2) H(t); \\ \dfrac{dP(t)}{dt} = \theta_1 H(t) - (\mu + \beta_1 + \beta_3) P(t); \\ \dfrac{dE(t)}{dt} = \theta_2 H(t) - (\mu + (1 - u_4(t))\gamma) E(t); \\ \dfrac{dD(t)}{dt} = \beta_1 P(t) + (1 - u_4(t))\gamma E(t) - (1 - u_2(t))\alpha_1 \dfrac{D(t)E(t)}{N} - \\ \qquad\qquad - (\mu + \beta_2 + \eta_2) D(t) + u_1(t) C_T(t); \\ \dfrac{dC_T(t)}{dt} = \beta_3 P(t) + \beta_2 D(t) + (1 - u_2(t))\alpha_1 \dfrac{D(t)E(t)}{N} - \\ \qquad\qquad - (\mu + \eta_1 + u_1(t))\alpha_2 \dfrac{C_T(t)E(t)}{N} - (\mu + \eta_1 + u_1(t)) C_T(t); \\ \dfrac{dC_S(t)}{dt} = \eta_2 D(t) + \eta_1 C_T(t) + (1 - u_3(t))\alpha_2 \dfrac{C_T(t)E(t)}{N} - (\mu + \delta_1) C_S(t), \end{cases}$$

where the controls $u_1(t)$, $u_2(t), u_3(t)$, and C are the proposed controls.

The objective function we are trying to minimize is given by :

$$J(u_1, u_2, u_3, u_4) = C_T(T_f) - D(T_f) - E(T_f) +$$

$$+ \int_0^{T_f} \left[C_T(t) - D(t) - E(t) + \frac{A}{2} u_1^2(t) + \frac{B}{2} u_2^2(t) + \frac{F}{2} u_3^2(t) + \frac{G}{2} u_4^2(t) \right] dt,$$

where $A > 0$, $B > 0$, $F > 0$, and $G > 0$.

The system (4) is well defined, moreover theorems 1, 2, 3, 4, and 5 in [11] have shown respectively the existence and uniqueness and the positivity of the solution, the existence of an optimal control and the characterization of this control for this system (4):

$$\begin{cases} \dfrac{(u_1, u_2, u_3, u_4)}{0} \le u_{1min} \le u_1(t) \le u_{1max} \le 1; \\ 0 \le u_{2min} \le u_2(t) \le u_{2max} \le 1; \\ 0 \le u_{3min} \le u_3(t) \le u_{3max} \le 1; \\ 0 \le u_{4min} \le u_4(t) \le u_{4max} \le 1 \quad / t \in [0, T_f]. \end{cases} \tag{3}$$

Characterisation of the control optimal. The principle of maximum Pontryagin and Hamiltonian has been used to characterize the optimal control, and the adjoint variables. After mathematical calculations and demonstrations that can be found in [11], we find that the adjoint variables are defined by:

$$\lambda_1' = \lambda_1(\mu + \theta_1 + \theta_2) - \lambda_2\theta_1 - \lambda_3\theta_2,$$

$$\lambda_2' = \lambda_2(\mu + \beta_1 + \beta_3) - \lambda_4\beta_1 - \lambda_5\beta_3,$$

$$\lambda_3' = 1 + \lambda_3(\mu + \gamma(1 - u_4(t))) - \lambda_4\left[\gamma(1 - u_4(t)) - \alpha_1(1 - u(t))\frac{D(t)}{N}\right] -$$

$$-\lambda_5\left[\alpha_1(1 - u_2(t))\frac{D(t)}{N} - \alpha_2(1 - u_3(t))\frac{C_T(t)}{N}\right] - \lambda_6\left[\alpha_2(1 - u_3(t))\frac{C_T(t)}{N}\right],$$

$$\lambda_4' = 1 - \lambda_4\left[\left(\alpha_1(1 - u_2(t))\frac{E(t)}{N} + (\mu + \beta_2 + \eta_2)\right)\right] -$$

$$-\lambda_5\left[\beta_2 + \alpha_1(1 - u_2(t))\frac{E(t)}{N}\right] - \lambda_6\eta_2, \qquad (4)$$

$$\lambda_5' = -1 - \lambda_4 u_1(t) + \lambda_5\left[-\alpha_2(1 - u_3(t))\frac{E(t)}{N} - (\mu + \eta_1 + u_1(t))\right] -$$

$$-\lambda_6\left[\eta_1 + \alpha_2(1 - u_3(t))\frac{E(t)}{N}\right],$$

$$\lambda_6' = \lambda_6(\mu + \delta).$$

With the transversality conditions at time T_f, $\lambda_1(T_f) = 0$, $\lambda_2(T_f) = 0$, $\lambda_3(T_f) = 1$, $\lambda_4(T_f) = 1$, $\lambda_5(T_f) = -1$ and $\lambda_6(T_f) = 0$.

Furthermore, for $t \in [0, T_f]$, the optimal controls u_1^*, u_2^*, u_3^* and u_4^* are given by:

$$u_1^* = \min\left(1, \max\left(0, \frac{(\lambda_5 - \lambda_4)}{A}C_T(t)\right)\right),$$

$$u_2^* = \min\left(1, \max\left(0, \alpha_1 \times \frac{\lambda_5 - \lambda_4}{B} \times \frac{D(t)E(t)}{N}\right)\right),$$

$$u_3^* = \min\left(1, \max\left(0, \alpha_2 \times \frac{\lambda_6 - \lambda_5}{F} \times \frac{C_T(t)E(t)}{N}\right)\right),$$

$$u_4^* = \min\left(1, \max\left(0, \frac{(\lambda_4 - \lambda_3)}{G} \times \gamma E(t)\right)\right).$$

EXPERIMENTAL RESULTS

In this section, we represent a comparison between the classical numerical method used to solve the mathematical models proposed in the literature and the intelligent genetic algorithm method. In the first part we will start with the kernel model (1) and in the second part we will make the comparison in the nonlinear model (4).

In this research paper, we used the genetic algorithm to adjust the different controls and the different parameters of the system, knowing that it is a powerful evolutionary algorithm.

The advantage of using the genetic algorithm to solve this type of optimal control problem is that we do not need to determine the adjoint values or characterize the control in any specific way. We only need to treat the problem well and specify the objective function we need to minimize and its various constraints. We present the results obtained by solving numerically the optimality systems (1) and (4) using the genetic algorithm method. In this control problems, we used the (GA) to find the most accurate initial conditions for the state variables.

In both models, which we will study, the quality of the controls of each of them, we have used the same values of the parameters that were used in the work [11; 13]. MatLab was used to write and compile the code, and the following data was used:

Genetic algorithm configuration:

Crossover : multiple;

Crossover : 0.8;

Initialization : random;

Number of iteration: $100 \times dim$;

Mutation : gaussian;

Population size : 200;

Selection function : stochastic(uniform).

The choice of the parameters of the genetic algorithm operators (selection, crossover, mutation) is a difficult task. In our work, our configuration is based on the following information:

a) Crossover: this step allows the production of new solutions, from the previous individuals. Each bit is chosen from either parent with the same probability [35; 36].

b) Mutation: it is sometimes found that all solutions are weak, which means the crossover operator does not lead to a new gene. The mutation causes random changes of rate pm in the genes, but this rate should not be larger, to avoid loss of the principle of selection and evolution [37].

c) Population size: the size of the population has a direct influence on the optimal control capability of our problem. Our choice of this parameter is based on [38; 39].

Linear EDC model

Let's start with the linear model (1). The use of single-objective optimization methods is not obvious on all this type of problem, first of all the understanding of the phenomenon is essential to put the pseudo-code simple to execute. Secondly, for a very good choice of the initial population that an important step in the method of genetic algorithm.

The Boutayeb model (1) parameters: $I = 2000000; \mu = 0.02;\ \gamma = 0.08;\ \nu = 0.05;$ $\delta = 0.05; \beta_1 = 0.5; \beta_2 = 0.1; \beta_3 = 0.5;\ \ dim = 0 : .1 :10; A = 3550000.$

The Boutayeb model (2) compartiments initialization: $E(0) = 66.6 \times 105;$ $D(0) = 102 \times 105;\ C(0) = 55 \times 105.$

As the linear case [33; 34], we used least square linear method to estimate the values of the parameters of the dynamic system of population diabetic and we found values close to those of the research paper [13; 34], which was based on the values of the World Diabetes Federation [1; 4].

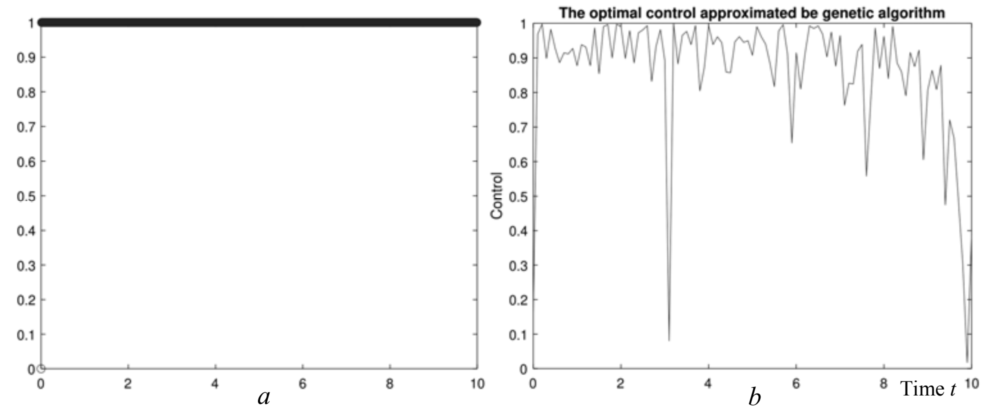

Fig. 1. The comparison between the control given by the GA and Gumel methods: *a* — the control *u* given by system (1) using Gumel method; *b* — the control *u* using GA

From the Fig. 1, *a*, we can notice that the use of the Gauss–Seidel type implicit finite difference method developed by Gumel [19] is give an estimate and does not give an optimal control $u*$.

For that we propose to solve this problem with the genitic algorithm (GA), we can see in the Fig. 2, *a*, *b*, *c* and Fig. 1, *b* that GA gives an optimal control but does not give a good effect.

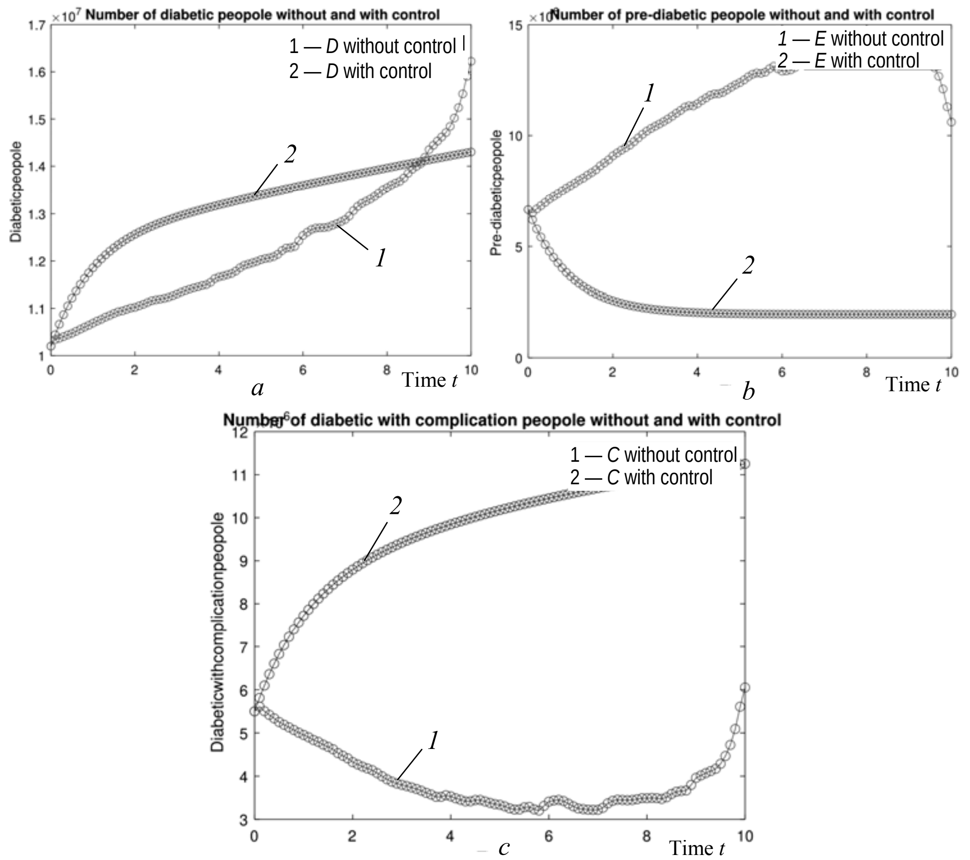

Fig. 2. The compartment *E*, *D* and *C* with and without control: *a* — the compartment *D* with and without control using GA; *b* — the compartment *E* with and without control using GA; *c* — the compartment *C* with complications withand without control using GA

It can be noticed from Fig. 1, *a* that the control given by Gumel's method takes almost a fixed value which is exactly 1. Of course, this allows a com- plete control of the dynamics of the phenomenon, throughout the five years, but consumes the totality of human and material resources. The proposed solution, based on a nonlinear model and a genetic algorithm, produces reasonable controls with very low cost (in fact, we never need all the resources in a given period to achieve our goal) (see Fig. 2). Furthermore, the genetic algorithm shows a great ability to produce controls in only a few iterations.

A nonlinear $HP\,EDC_SC_T$ model

We now turn to the non-linear model (4) which contains 6 compartments. We have seen that four compartments have been controlled by four different controls. Our objective is to learn about the controls given by the intillegent method and their quality.

The model 4 shows that the number of pre-diabetic population by the negative effect of behavioral factors on diabetic patients and other diabetics *E* and *D* without complications are those who control blood glucose by diet and exercise before it is too late, C_T diabetics with treatable complication and C_S diabetics with severe complications is decreased by the use of controls u_1, u_2, u_3 and u_4 respectively. This can be seen clearly in the Figs. 3, *a*, *b*, 4, *a*, *b*, 5, *a*, *b*.

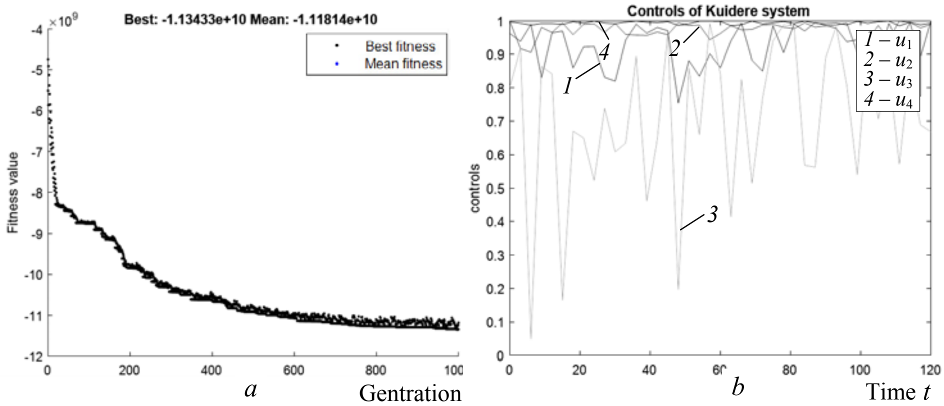

Fig. 3. The controls of non-linear system: *a* — objective function of GA; *b* — the controls of non-linear system

In genetic algorithm, there exist several types of convergences: a) the constructed control sequence becomes very close to the optimal control sought; b) the constructed control sequence becomes stagnant; c) the cost sequence of the constructed control sequence becomes stagnant; d) the maximum number of iterations, set by the user, is exhausted. It is impossible to refer to the optimal control sought because it is not known; therefore the first type of convergence remains impractical. The second and third type of convergence is directly influenced by the size of the population, the crossover rate, and the mutation rate; indeed, a bad choice of these can lead the genetic search to very bad local minima. The fourth type of convergence is called artificial convergence because the user is satisfied with a reasonable improvement of the quality of the initial population especially when the studied phenome is very complex and calls for several algorithmic components. We opted for the artificial convergence because the studied system is very complex and the control estimation is part of a very large research project involving several components. Moreover, we can notice that a few iterations were able to achieve a very good estimation; of course, we can increase the number of iterations to improve the obtained control even if the one obtained is very satisfactory (see Fig. 3, *a*).

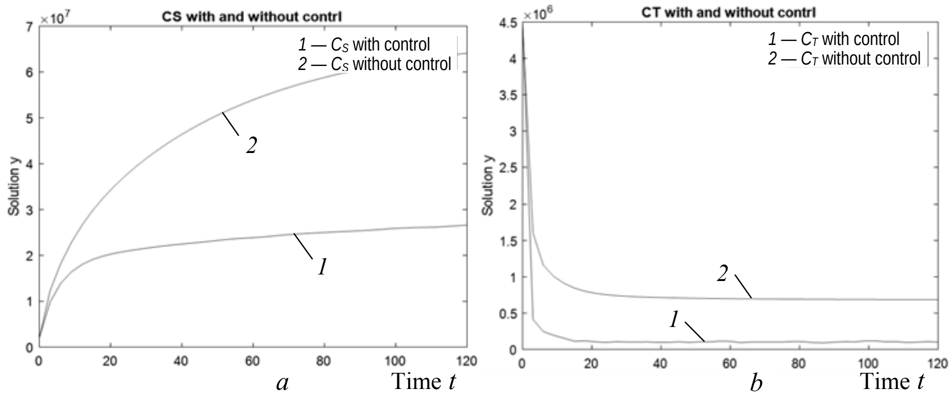

Fig. 4. The compartment (*a*) C_T and (*b*) C_S with and without control given by system (7) and (10) using GA

In this work, we present two types of models. A linear dynamic model and a non-linear model. In the first model, we considered a single control on both behaviors, which does not correspond to human nature, because each diabetic patient has his own needs. But the second non-linear model takes into account 6 different behaviors, that is, the most possible cases of diabetes, moreover four controls were considered, which helps us to treat each behavior with its needs. In conclusion, the dynamic controlled model $HPEDC_TC_S$ gives us more infor- mation because of the diversity of diabetes types, and the control strategy for each type.

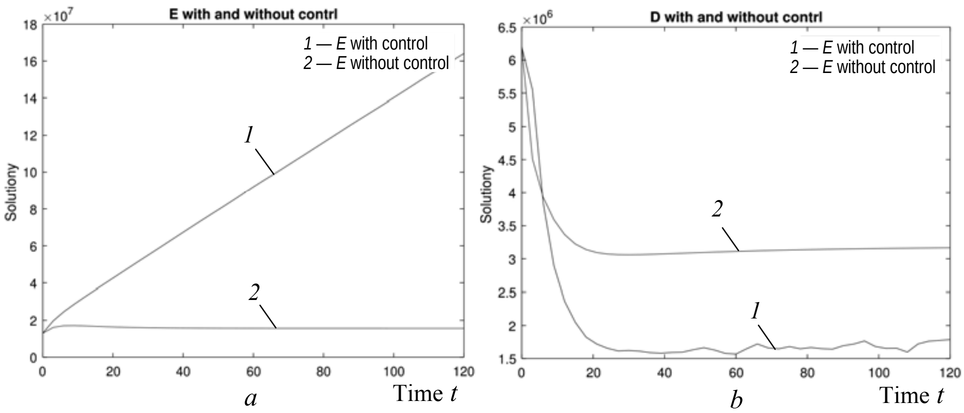

Fig. 5. The compartment E (*a*) and D (*b*) with and without control given by systems (2) and (3) using GA

In general, we introduced a comparison between the following systems: a) Gumel and linear model; b) Gumel and non-linear model; and c) Genetic Algorithm and non-linear model.

Solved by Gumel method, the linear and non-linear models, the produced control recommends always the implementation of all disponible resources because of the formula of the control that implements the operators max-min which leads to the value 1. Solved by genetic algorithm, the linear and non-linear models, the produced control is capable to reduce significantly the number of diabetic and diabetics with complications using less than 70% (mean) of resources (see

Figs. 1 and 3). In addition, the non-linear system is more suitable to understand and control the phenomenon under study.

The estimated control are in the hand of diabetologists and it will be traduced in terms of medication, exercise, and diet.

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CONCLUSION

In this paper we have elaborated a state of the art on the different dynamic systems with economic function proposed in the literature controlling diabetes in order to alleviate the socio-economic damage caused by it. Then, we have used of the best performing local search artificial intelligence methods (metaheuristics) to solve the kernel model dealing with the common compartments between all these models. Controlling diabetes is a major challenge, and studies have highlighted the lim- itations of linear models in describing the complexity of this chronic disease. Non-linear models, by introducing more realistic terms, have enabled a better understanding of the dynamics of the diabetic population. However, traditional methods of order estimation, such as Pantryagain's maximum principle and Gumel's numerical method, have proved costly and difficult for diabetologist professionals to implement. In this paper, we have proposed an innovative strategy based on nonlinear models and the use of GAs. This three-step approach discretizes the nonlinear model, estimates the optimal control using GAs, and constructs the final control using an interpolation model. The results obtained demonstrated the success of this approach, with a significant reduction in the number of cases of diabetes and complications in diabetic patients, while using less than 70% of available resources. Despite the success of GAs in diabetes control, there are a number of limitations to consider. Firstly, model resolution time can be a challenge, not least due to the complexity of the non-linear models used. In addition, the use of integer derivatives may restrict access to certain relevant information. In future work, it is proposed to explore two avenues to overcome these limitations, 1) the use of parallelism techniques could speed up model solution time; 2) the use of fractional equations rather than ordinary differential equations could be considered.

REFERENCES

1. "Definition and diagnosis of diabetes mellitus and inter mediate hyperglycaemia," *World Health Organisation (WHO), Geneva, 2016*.
2. A. El Ouissari, K. El Moutaouakil,"Density based fuzzy support vector machine: application to diabetes dataset," *Mathematical Modeling and Computing*, vol. 8, no. 4, pp. 747–760, 2021.
3. K. El Moutaouakil, M. Cheggour, S. Chellak, and H. Baizri, "Meta- heuristics Optimization Algorithm to an Optimal Moroccan Diet," in *2021 7th*

Annual International Conference on Network and Information Systems for Computers(ICNISC), IEEE, 2021, pp. 364–368.
4. *IDF DIABETES ATLAS;* 9th edition. International Diabetes Federation (IDF), 2019.
5. A. Boutayeb, E.H. Twizell, K. Achouayb, and A. Chetouani, "A mathematical model for the burden of diabetes and its complications," *Biomedical engineering online*, 3(1), pp. 1–8, 2004.
6. A. Boutayeb, A. Chetouani, K. Achouayb, and E.H. Twizell, "A non- linear population model of diabetes mellitus," *Journal of Applied Mathematics and Computing*, 21(1), pp. 127–139, 2006.
7. A. Nakonechnyj, V.Marzenyuk, "Problems of controllability for differential Gompertzian dynamics equations," *Cybernetics and Systems Analysis*, 40(2), pp. 123–133, 2004.
8. K. Marti, Y. Ermoliev, M. Makowski, G. Pflug, A.G. Nakonechny, and V.P. Marzeniuk, "Uncertainties in medical processes control," in *Coping with Uncertainty: Modeling and Policy Issues*, pp. 185–192. Springer Berlin Heidelberg, 2006.
9. A. Kouidere, O. Balatif, H. Ferjouchia, A. Boutayeb, and M. Rachik, "Optimal control strategy for a discrete time to the dynamics of a population of diabetics with highlighting the impact of living environment," *Discrete Dynamics in Nature and Society*, 2019.
10. A. Boutayeb, A. Chetouani, "A population model of diabetes and pre-diabetes," *International Journal of Computer Mathematics*, 84(1), pp. 57–66, 2007.
11. A. Kouidere, A. Labzai, H. Ferjouchia, O. Balatif, and M. Rachik, "A New Mathematical Modeling with Optimal Control Strategy for the Dynamics of Population of Diabetics and Its Complications with Effect of Behavioral Factors," *Journal of Applied Mathematics*, 2020.
12. A. Kouidere, B. Khajji, O. Balatif, and M. Rachik, "A multi-age math-ematical modeling of the dynamics of population diabetics with effect of lifestyle using optimal control," *Journal of Applied Mathematics and Computing*, vol. 67, pp. 375–403, 2021.
13. M. Derouich, A. Boutayeb, W. Boutayeb, and M. Lamlili, "Optimal control approach to the dynamics of a population of diabetics," *Applied Mathematical Sciences*, 8(56), pp. 2773–2782, 2014.
14. K. El Moutaouakil, M. Roudani, and A. El Ouissari, "Optimal Entropy Genetic Fuzzy-C-Means SMOTE (OEGFCM-SMOTE)," *Knowledge-Based Systems*, 262, 110235, 2023.
15. A. Mahata, S.P. Mondal, S. Alam, A. Chakraborty, S.K. Dey, and A. Goswami, "Mathematical model for diabetes in fuzzy environment with stability analysis," *Journal of Intelligent and Fuzzy Systems*, 36(3), pp. 2923–2932, 2019.
16. R.L. Ollerton, "Application of optimal control theory to diabetes mellitus," *International Journal of Control*, 50(6), pp. 2503–2522, 1989.
17. G.W. Swan, "An optimal control model of diabetes mellitus," *Bulletin of Mathematical Biology*, 44(6), pp. 793–808, 1982.
18. A. Makroglou, I. Karaoustas, J. Li, and Y. Kuang, "Delay differential equation models in diabetes modelling," *Theoretical Biology and Medical Modelling*, 2009.
19. A.B. Gumel, P.N. Shivakumar, and B.M. Sahai, "A mathematical model for the dynamics of HIV-1 during the typical course of infection," in *Proceedings of the 3rdWorld Congress of Nonlinear Analysts*, 47 (2011), pp. 2073–2083.
20. F. Hayes-Roth, Review of "Adaptation in Natural and Artificial Systems by John H. Holland", The U. of Michigan Press, 1975, *ACM SIGART Bulletin*, issue 53, pp. 15, 1975.
21. L.B. Booker, D.E. Goldberg, and J.H. Holland, "Classifier systems and genetic algorithms," *Artificial Intelligence*, 40(1-3), pp. 235–282, 1989.
22. K. Krishnakumar, D.E. Goldberg, "Control system optimization using genetic algorithms," *Journal of Guidance, Control, and Dynamics*, 15(3), pp. 735–740, 1992.

23. Z. Michalewicz, C.Z. Janikow, and J.B. Krawczyk, „A modified genetic algo- rithm for optimal control problems," *Computers & Mathematics with Applications*, 23(12), pp. 83–94, 1992.
24. N. Marco, C. Godart, J.A. Desideri, B. Mantel, and J. Periaux, *A genetic algorithm compared with a gradient-based method for the solution of an active- control model problem (Doctoral dissertation, INRIA*), 1996
25. I.C. Lerman, R. Ngouenet, *Algorithmes genetiques sequentiels et paralleles pour une representation affine des proximites (Doctoral dissertation, INRIA),* 1995.
26. L. Rarita, I. Stamova, and S. Tomasiello, "Numerical schemes and genetic algorithms for the optimal control of a continuous model of supply chains," *Applied Mathematics and Computation*, 388, 125464, 2021.
27. D. Castaldo, M. Rosa, and S. Corni, "Quantum optimal control with quantum computers: A hybrid algorithm featuring machine learning optimization," *Physical Review A*, 103(2), 022613, 2021.
28. J. Wang, J. Hou, J. Chen, Q. Fu, and G. Huang, "Data mining approach for improving the optimal control of HVAC systems: An event-driven strategy," *Journal of Building Engineering*, 39, 102246, 2021
29. H.H. Mehne, S. Mirjalili, "A parallel numerical method for solving optimal control problems based on whale optimization algorithm," *Knowledge-Based Systems*, 151, pp. 114–123, 2018.
30. A.K. John, K. Krishnakumar, "Performing multiobjective optimization on perforated plate matrix heat exchanger surfaces using genetic algorithm," *International Journal for Simulation and Multidisciplinary Design Optimization*, 8, A3, 2017.
31. I.M. Fanuel, A. Mushi, and D. Kajunguri, "Irrigation water allocation optimization using multi-objective evolutionary algorithm (MOEA) a review," *International Journal for Simulation and Multidisciplinary Design Optimization*, 9, A3, 2018.
32. N.R. Nagaiah, C.D. Geiger, "Application of evolutionary algorithms to optimize cooling channels," *International Journal for Simulation and Multidisciplinary Design Optimization*, 10, A4, 2019.
33. K. El Moutaouakil, A. El Ouissari, B. Hicham, C. Saliha, and M. Cheggour, "Multi-objectives optimization and convolution fuzzy C-means: control of diabetic population dynamic," *RAIRO-Operations Research*, 56(5), pp. 3245–3256, 2022. doi: 10.1051/ro/2022142.
34. Abdellatif el Ouissari, Karim el Moutaouakil, Baizri Hicham, and Chellak Saliha, "Intelligent Local Search for an Optimal Control of Diabetic Population Dynamics," *Mathematical Models and Computer Simulations*, 14, pp. 1051–1071, 2022. doi: 10.1134/S2070048222060047.
35. Darrell Whitley, "A genetic algorithm tutorial," *Statistics and computing*, vol. 4, no. 2, pp. 65–85, 1994,
36. David E. Goldberg, *Genetic Algorithm in Search, Optimization and Machine Learning*. New Jersey, Addison-Wesley, 1989.
37. A.E. Eiben, J.E. Smith, *Introduction to evolutionary computing*. Berlin: Springer, 2003.
38. Masaaki Horie, Naoki Fukuta, "An Evolutionary Approach for Simulating Contin- uous Coalition Formation", *2018 7th International Congress on Advanced Applied Informatics (IIAI-AAI)*, pp. 588–593.
39. B.R. Rajakumar, A. George, "APOGA: An adaptive population pool size based genetic algorithm," *AASRI Procedia*, 4, pp. 288–296, 2013.
40. V.P. Martsenyuk, I.Y. Andrushchak, and A.M. Kuchvara, "On conditions of asymptotic stability in SIR-models of mathematical epidemiology," *Journal of Automation and Information Sciences*, 43(12), 2011.

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ІНТЕЛЕКТУАЛЬНЕ ОПТИМАЛЬНЕ КЕРУВАННЯ НЕЛІНІЙНОЮ СИСТЕМОЮ ПОПУЛЯЦІЙНОЇ ДИНАМІКИ ХВОРИХ НА ДІАБЕТ ІЗ ВИКОРИСТАННЯМ ГЕНЕТИЧНОГО АЛГОРИТМУ / Абделлатиф Ель Уіссарі, Карім Ель Мутауакіль

Анотація. Цукровий діабет є хронічним захворюванням, яким страждають мільйони людей у всьому світі. Виконано кілька досліджень, спрямованих на контроль проблеми діабету, із використанням як лінійних, так і нелінійних моделей. Однак складність лінійних моделей не в змозі глибинно описати динаміку діабетичного населення. Щоб отримати більше деталей про цю динаміку, до математичних моделей уведено нелінійні терміни, що призвело до більш складних моделей, які повністю відповідають реальності (здатні відтворювати спостережувані дані). Найбільш часто використовуваними методами для оцінювання контролю є принцип максимуму Пантрягейна та числовий метод Гумеля. Однак ці методи призводять до дуже дорогої стратегії з точки зору матеріальних і людських ресурсів; крім того, діабетологи не в змозі використовувати формули, реалізовані запропонованими елементами контролю. Запропоновано вибіркову стратегію та продуктивність, засновану на нелінійних моделях і генетичних алгоритмах (GA), яка виконується в три етапи: 1) дискретизація розглянутої нелінійної моделі за допомогою класичних числових методів (правило трапеції та алгоритм Ейлера–Коші); 2) оцінювання оптимального контролю в кількох точках на основі GA з відповідною функцією пристосованості та відповідними генетичними операторами (мутація, схрещування та відбір); 3) побудова оптимального керування за допомогою інтерполяційної моделі (сплайнів). Отримані результати показали, що використання GA для нелінійних моделей було успішно вирішено, що призвело до контрольного підходу, який демонструє значне зменшення кількості випадків діабету та діабетиків з ускладненнями. Примітно, що цей результат досягається з використанням менше ніж 70% доступних ресурсів.

Ключові слова: оптимальне керування, диференціальне рівняння, діабет, генетичні алгоритми, штучний інтелект, інтелектуальний локальний пошук.