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EXPANSION OF THE MATHEMATICAL APPARATUS OF DISCRETE-CONTINUOUS NETWORKS FOR THE AUTOMATION OF THEIR SYNTHESIS PROCEDURES

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Abstract. The paper deals with a model of an intelligent system related to the automatic synthesis of Petri nets and presents a certain stage of developing this model. The peculiarity of the extended mathematical apparatus is that it contains a combination of Petri net incidence matrices to represent various algorithms. This combination of matrices is part of the equations describing the logic control device of a complex system. Accordingly, the work also presents a well-known mathematical description of discrete-continuous systems with a controlled structure, which includes certain logical control devices. This mathematical description, based on means of discrete-continuous networks, is associated with the incidence matrix of the Petri net, which is formed as a result of a particular synthesis algorithm. At the same time, the formed Petri net represents the corresponding logical control algorithm that should ensure the effective functioning of the corresponding system. The final part of the work presents various structural schemes of logic-dynamic models of systems related to the automatic synthesis of Petri nets. Here, we determine the features of the advanced mathematical apparatus based on discrete-continuous networks to develop an intelligent system that forms logical control algorithms. It is also noted that such systems can be used to create certain control algorithms that ensure increased efficiency of the functioning of some objects in difficult and unpredictable conditions.

Keywords: Petri nets, system with controlled structure, discrete-continuous network, automatic synthesis of Petri nets.

INTRODUCTION

Petri nets, as an applied mathematical apparatus, are quite well-known in the field of modeling and analysis of discrete dynamic or logic-dynamic systems. Petri nets were first proposed by Carl Adam Petri in 1962 as part of his dissertation work – “Communication with automata”. Petri's work became a significant contribution to the development of parallel and distributed computing. Such a concept as the automatic synthesis of Petri nets can be found in the work of James Peterson [1] as a direction related to the development of certain algorithms. At the same time, the development of methods for the automatic synthesis of Petri nets entails the need to expand the mathematical apparatus for describing complex systems, functioning algorithms that can be represented by Petri nets.

REVIEW OF DISCRETE-CONTINUOUS NETWORKS

Nowadays, Petri nets are greatly expanded. So, for example, there are many varieties of Petri nets, such as: time Petri nets, inhibitor Petri nets, colored Petri nets, hybrid Petri nets, and others [2; 3].

The history of the corresponding scientific direction and the corresponding scientific thought, starting with Carl Adam Petri, is quite long, so there is no need to consider separate stages of development or less important elements. But, in this case, it is necessary to note the invention of a discrete-continuous network [4]. Discrete continuous network (DC-net) proposed in 1990–1993, is essentially a synthesis of structural schemes of automatic control systems and Petri nets, which are not the usual extension such as, for example, hybrid Petri nets. DC-net is primarily a tool for describing, modeling and analyzing logic-dynamic systems and systems with a controlled structure.

The description and modeling of systems by means of discrete-continuous networks allows us to imagine a certain class of systems called discrete-continuous with a controlled structure (DCCS). In English-language publications, such systems are called hybrid systems, and both traditional Petri nets and their variants, in individual cases, hybrid Petri nets are used to study such systems [5; 6].

DC-nets, like Petri nets, is an applied mathematical apparatus and we extent it in order to develop the technique of automatic synthesis of Petri nets.

The development of models based on such an advanced mathematical apparatus will allow solving complex problems related to the development of certain algorithms. As an example, it is worth noting the so-called “smart ant” problem, which is presented in works [7; 8]. An ant builds an automaton of its behavior with the help of trial and error and mutations. Thus, assume that a model built with the use of DC-nets, can synthesize an automaton of its behavior or an algorithm of some logical control; then we advise to use such a model at the stage of automated development of control algorithms, control systems, or as a certain intelligent system [9].

So, it can be noted that in this case it is necessary to expand the mathematical apparatus while developing methods of automatic synthesis of Petri nets. The paper is relevant due to the development of certain systems that provide the synthesis of Petri nets with the use of modern intelligent technologies such as fuzzy logic, artificial neural networks, genetic algorithms, etc. [10; 11].

Purpose of work: The purpose of this work is to minimize time and automate the process of synthesis of some control algorithms of complex systems.

To achieve the goal, we expand the mathematical apparatus of discrete-continuous networks, taking into account the procedure of automatic synthesis of Petri nets.

MAIN PART

Description of the system with a controlled structure

Modeling tools of DC-nets allow to present a model of complex technical systems consisting of two parts: continuous-event part (CEP) and discrete-event part (DEP) in a structural unity. Such a system was called a system with a controlled structure (SCS). The continuous-event part of the model represents the control object with a controlled structure (COCS) and the DEP of DC-net represents the logical control device (LCD).

The COCS is represented by the state and output equations:

$$\dot{x}(t) = f(\Xi(t_k), x(t), u(t)); \quad (1)$$

$$y(t) = f(\Xi(t_k), x(t)), \quad (2)$$

where $u(t)$ is continuous control vector; $x(t)$ is state vector; $y(t)$ is output vector; $\Xi(t_k)$ is vector function for controlling the structure of COCS (functioning modes). Accordingly, in such a system it is possible to identify a generalized input effect:

$$U = (u(t), \Xi(t_k)).$$

The LCD is represented by a finite state machine (3),(4) characterized by the equations:

$$a_k = \lambda(a_{k-1}, \upsilon_k), \quad (3)$$

$$\beta_k = \gamma(a_k, \beta_k), \quad (4)$$

where $A = \{a_1, a_2, ..., a_k, ...a_i\}$ is a finite set of internal states, $\{\upsilon_1, \upsilon_2, ..., \upsilon_k, ..., \upsilon_j\}$ is an iput alphabet, $\{\beta_1, \beta_2, ..., \beta_k, ..., \beta_v\}$ is an output alphabet, λ is a transition function (from state to another state), γ is an output function.

The presented formal form of equations (1)–(4) is appropriate without taking into account the procedure for automatic synthesis of LCD control algorithm and, accordingly, the automatic generation of a Petri net during the operation of SCS.

Mathematical description of discrete-continuous systems with a controlled structure with the use of the DC-net

The mathematical description of a discrete-continuous system with a controlled structure, taking into account the means of DC-net, can be obtained from a set of equations.

Dynamics of COCS in continuous space $X(t_{,}|t_k|)$ can be represented in matrix-differential form by the equation of state:

$$\dot{X}(t_{,}|t_k|) = A_o \cdot \Xi_1({}^d u_o(t_k)) \cdot X(t_{,}|t_k|) + B_o \cdot \Xi_2({}^d u_o(t_k)) \cdot u(t_{,}|t_k|), \quad (5)$$

output equation

$$Y(t_{,}|t_k|) = C_o \cdot \Xi_3({}^d u_o(t_k)) \cdot X(t_{,}|t_k|), \quad (6)$$

equation of state of the LCD

$${}^d X_L(t_k) = {}^d X_L(t_{k-1}) + |A_L| \cdot {}^d v_L(t_k) + {}^d u_L(t_k) + {}^d w_L(t_k), \quad (7)$$

and the LCD output equation

$${}^d Y_L(t_k) = \Lambda \cdot {}^d X_L(t_k), \quad (8)$$

where $X(t_{,}|t_k|)$ is vector of continuous event state of COCS; $Y(t_{,}|t_k|)$ is continuous event output vector; $u(t_{,}|t_k|)$ is continuous exposure vector; $A_0 = \left| A_1^0 \quad A_2^0 \quad ... \quad A_N^0 \right|$; $B_0 = \left| B_1^0 \quad B_2^0 \quad ... \quad B_N^0 \right|$; $C_0 = \left| C_1^0 \quad C_2^0 \quad ... \quad C_N^0 \right|$; $A_1^0, A_2^0 ... A_N^0$, $B_1^0, B_2^0 ... B_N^0$ and $C_1^0, C_2^0 ... C_N^0$ — matrices of states, controls and outputs of different structural operating modes; $\Xi({}^d u_o(t_k))$ — vector function for managing structural changes (functioning modes of SCS); $\Xi_1({}^d u_o(t_k)) = = \left|\xi_1^1 \quad \xi_2^1 \quad \ldots \quad \xi_N^1\right|^{\mathrm{T}}$ and $\Xi_2({}^d u_o(t_k)) = \left|\xi_1^2 \quad \xi_2^2 \quad \ldots \quad \xi_N^2\right|^{\mathrm{T}}$ is vector-functions of structure control depending on the discrete state $X_0^d(t_k)$ continuous event part. They implement the selection of a specific structure from a variety of structures $\{\Sigma_i\}_{i=1}^N$ using matrix multiplication A_0, B_0, C_0. Vector function control $\Xi({}^d u_o(t_k))$ is a matrix of dimension $n \times 1$, that contains only one non-zero element. The dimension of the control function vector is consistent with the dimension of the matrices A_0, B_0, C_0; ${}^d u_o(t_k)$ is a discrete component of the control influence on CEP, transferring the system from one structure to another; ${}^d w_L(t_k)$ is external control action; ${}^d X_L(t_k)$, ${}^d X_L(t_{k-1})$ — preliminary and subsequent discrete state (labeling) of the Petri net; $|A_L|$ is incidence matrix reflecting the relationship of elements in DC-net; ${}^d v_L(t_k)$ is control vector in DC-net. A simplified block diagram of the SCS, according to the given mathematical description, is presented in Fig. 1.

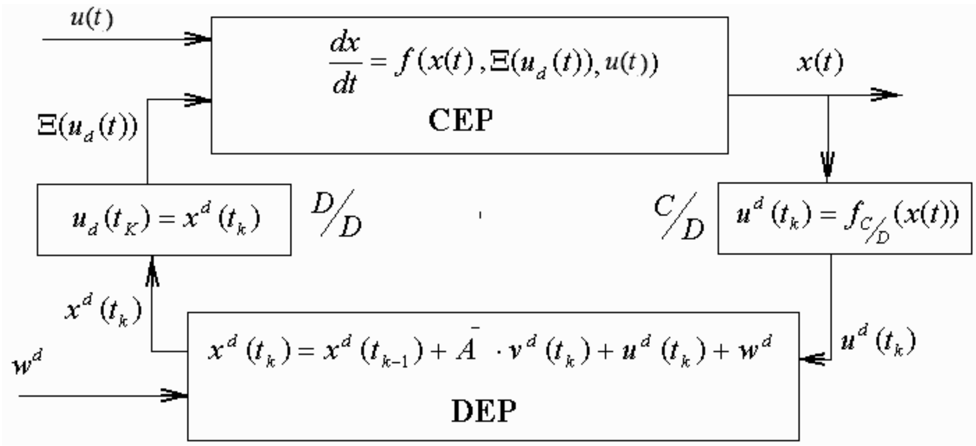

Fig. 1. Simplified block diagram of the logic-dynamic model

Expansion of the mathematical apparatus for describing complex systems based on procedures for automatic synthesis of Petri nets

These equations (5)–(8) are the basis for expanding the mathematical apparatus taking into account the procedures for automatic synthesis of Petri nets.

During the automatic synthesis of a Petri net, the dimension of vectors ${}^d X_L(t_k)$, ${}^d u_L(t_k)$ and matrices $|A_L|$ may not change, but the elements of the matrix must change during different runs of the system model. In this case, the incidence matrix $|A_L|$ may be $|A_{L1}|$, $|A_{L2}|$. Thus $|A_L| \in |A_{L1}|, |A_{L2}|, \ldots, |A_{Ln}|$, in this case, the value of n is unknown in advance and depends on the task at hand.

Matrix selection $|A_{Li}|$, where $i=1,\ldots,n$, depends on initial conditions $\overline{S}$, where $\overline{S}=[s_1 \quad s_2 \quad \ldots \quad s_n]^{\mathrm{T}}$ — a vector of initial conditions generated by some expert system. Taking into account the automatic synthesis of the Petri net, equation (7) will take the following form:

$$^{d}X_L(t_k)={}^{d}X_L(t_{k-1})+(W_L\cdot{}^{d}S({}^{d}u_S(t_k)))^{d}\cdot v_L(t_k)+{}^{d}u_L(t_k)+{}^{d}w_L(t_k)\,, \quad (9)$$

where $W_L=[|A_{L1}| \quad |A_{L2}| \quad \ldots \quad |A_{Ln}|]$; according $W_L\cdot{}^{d}S({}^{d}u_S(t_k))=|A_{Li}|$, where $i=1,\ldots,n$, ${}^{d}S({}^{d}u_S(t_k))=[s_1 \quad s_2 \quad \ldots \quad s_n]^T$, $\bigcup_{i=1}^{n}s_i=1$, $\bigcap_{i=1}^{n}s_i=0$.

Vector function $S({}^{d}u_S(t_k))$ contains elements

$$s_i=\begin{cases}1 & \text{at} \quad {}^{d}u_S(t_k)={}^{d}u_S(t_k)_{given};\\ 0 & \text{at} \quad {}^{d}u_S(t_k)\neq{}^{d}u_S(t_k)_{given},\end{cases}$$

where ${}^{d}u_S(t_k)=[{}^{d}u_{S1} \quad {}^{d}u_{S2} \quad \ldots \quad {}^{d}u_{Sm}]$; ${}^{d}u_{Si}=f_{C/D}(\Delta J(t),w)$; ${}^{d}u_S(t_k)_{given}$ — tasks vector; $\Delta J(t)=\int_{t_1}^{t_2}f(y,u,t)dt-\int_{t_3}^{t_4}f(y,u,t)dt$ is an/the increment of the system perfomance criterion; w — external influence from the expert; $f_{C/D}$ — function of continuous-discrete transformation of variables.

The equation (9) implies that the simplified block diagram presented in Fig. 1 is transformed as shown in Fig. 2.

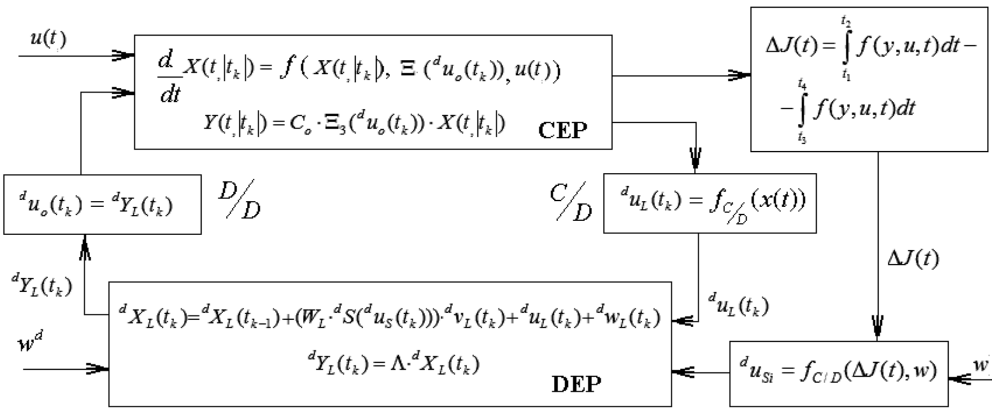

Fig. 2. Simplified block diagram of the logical-dynamic model of the system, implementing the automatic synthesis of Petri nets

To visualize the Petri net synthesis process, state variables ${}^{d}X_L(t_k)$, ${}^{d}X_L(t_{k-1})$ and increase in the value of the system performance criterion ΔJ can be output using a parametric file to the visualization platform, as shown in [12]. Visualization of the process of Petri net synthesis based on the corresponding Flash or Unity platform is an important component of the interaction of an expert with an intelligent system.

Research results

The consideration of the extended mathematical description of complex systems provides the basis for constructing a model of an intelligent system intended for the formation of some logical control algorithms.

These algorithms are formed according to certain methods through the automatic synthesis of Petri nets. Combination of different incidence matrices $|A_{Li}|$, where $i=1,\ldots,n$, that are included in equation (9) represents a set of various algorithms that can be adjusted during the functioning of the intelligent system. The block diagram presented in Fig. 2 can represent this intelligent system, taking into account the additional procedure for correcting the incidence matrix; $|A_{Li}|$ implemented, as shown in [11], taking into account the functioning of an artificial neural network and its training. Such a system can be used to form certain control algorithms that provide increased operating efficiency in some objects.

CONCLUSIONS

In this work we expand the mathematical description of complex technological systems with the use of DC-nets, taking into account the automatic synthesis of Petri nets.

Expanding the mathematical description of complex technological systems with the use of DC-nets makes it possible to approach the solution of a practical problem associated with the development of an intelligent system that automatically synthesizes some logical control algorithms.

In turn, the use of such an intelligent system makes it possible to achieve a certain goal of work, which is to minimize the time and material costs for the development of logical control algorithms.

Further scientific research should be related to the development of methods for the synthesis of Petri nets and some algorithms used in control systems.

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РОЗШИРЕННЯ МАТЕМАТИЧНОГО АПАРАТУ ДИСКРЕТНО-НЕПЕРЕРВНИХ МЕРЕЖ ДЛЯ АВТОМАТИЗАЦІЇ ПРОЦЕДУР ЇХ СИНТЕЗУ/ О.О. Гурський, А.В. Денисенко, О.Є. Гончаренко

Анотація. Подано певний етап розроблення моделі інтелектуальної системи, пов’язаної з автоматичним синтезом мереж Петрі. Розглянуто розширений математичний опис складних систем на основі засобів дискретно-неперервних мереж, який покладено в основу розроблення такої інтелектуальної системи, спрямованої передусім на формування алгоритмів логічного керування. Особливість розширеного математичного апарату полягає у тому, що у його складі є комбінація матриць інцидентності мереж Петрі для подання різноманітних алгоритмів. Ця комбінація матриць входить до складу рівнянь, що описує пристрій логічного керування складної системи. Відповідно подано відомий математичний опис дискретно-неперервних систем із керованою структурою, що включають певні пристрої логічного керування. Цей математичний опис на основі засобів дискретно-неперервних мереж, пов’язаний з матрицею інцидентності мережі Петрі, що формується в результаті певного алгоритму синтезу. Сформовано мережу Петрі — відповідний алгоритм логічного керування для забезпечення процесу ефективного функціонування відповідної системи. Подано різні структурні схеми логіко-динамічних моделей систем з автоматичним синтезом мереж Петрі. Визначено особливість розширеного математичного апарату на основі дискретно-неперервних мереж для розроблення інтелектуальної системи, що формує алгоритми логічного керування. Такі системи можна використовувати для формування певних алгоритмів керування, які забезпечують підвищену ефективність функціонування деяких об’єктів.

Ключові слова: мережі Петрі, система з керованою структурою, дискретно-неперервна мережа, автоматичний синтез мереж Петрі.