

NUMERICAL ALGORITHM FOR CALCULATION OF THE VACUUM CONDUCTIVITY OF A NON-LINEAR CHANNEL FOR TRANSPORTING A SHORT-FOCUS ELECTRON BEAM IN THE TECHNOLOGICAL EQUIPMENT

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Abstract. In the article, based on solving the equations of vacuum technology, an iterative algorithm for calculating vacuum conductivity and the geometric parameters of a curvilinear channel for transporting a short-focus electron beam is proposed and studied. For such a type of channel, the dependence of its radius on the longitudinal coordinate is described by a power function. The proposed algorithm is based on the numerical solution of a set of nonlinear equations using the Steffensen method. The results of the test calculations are presented. The provided tests confirm the stability of the proposed algorithm's convergence for correct pressure and pumping speed values in electron-beam technological equipment. Such curved transport channels can be used in electron beam equipment based on high-voltage glow discharge electron guns intended for welding, melting metals, and the deposition of thin films. The criterion for the optimal geometry of a nonlinear channel is the minimum power loss of the electron beam during its transportation while ensuring the required pressure drop between the discharge and technological chambers.

Keywords: electron beam, electron beam technologies, electron beam transportation, nonlinear electron beam transportation channel, vacuum conductivity of the transportation channel, high-voltage glow discharge electron gun, vacuum technology equation, set of nonlinear equations, Steffensen method.

INTRODUCTION

The development of electron beam technologies is very important today, and such advanced technologies are widely applied in different branches of industry, including metallurgy, mechanical engineering, energetic industry, electronics, instrument-making industry, automotive industry, as well as aircraft and space industry [1–44]. Generally, it is caused by the many important advantages of the electron beam as a technological instrument, which are as follows [1–10].

1. The total power and power density of the electron beam are extremely high. Naturally, in industrial technological electron guns, the total power can reach hundreds of kW, and the power density can be up to 10^9 W/m².
2. Ease of control and changing of the geometric and energy focal parameters of the electron beam using electric and magnetic fields.
3. Carrying out a technological operation under conditions of medium and high vacuum, which ensures the repeatability of technological process parameters during their control, the purity of the treated materials, and, as a result, the high quality of products.

In particular, electron beam methods for purifying refractory metals and ceramic materials are widely used today in metallurgy [36–41]. For example, with

the development of modern electronics, it is now very important to obtain pure silicon for use in electronics for the production of effective and high-quality microchips [36]. Advanced technologies for refining refractory metals are very important for the production of reliable details for the automotive, aircraft, and space industries [40; 41].

Recently, in the mechanical engineering, aviation, and space industries, electron beam technologies for three-dimensional metal printing have found wide application and are gradually becoming quite cheap, highly efficient, and allow the economical use of metal raw materials and electricity [21–25]. Such technologies also make it possible to obtain high-strength and reliable parts for the chemical industry, aviation, and space industries. Usually, the mechanical and chemical properties of metals produced using three-dimensional printing are unique, and it is really impossible to obtain metals of such quality using traditional metallurgy methods [42–44].

In the electronics industry, it is effective to use the technology of welding with point-focus [14–19] and profile [20–22] electron beams for sealing the housings of electronic devices and welding metal and ceramic contacts. For example, in papers [14; 15], the possibility of peer-reviewed welding of contacts of cryogenic electronic devices with a short-duration pulsed point-focus electron beam has been considered [16–19]. Profile beam welding is a very low-cost technology that provides high productivity and can be easily automated [20–22].

Another effective use of electron beam technologies in modern production is the application of stoichiometric ceramic coatings, which contain active gases, in particular oxides, carbides, sulfides, nitrides, etc. [23–32]. For example, multi-layer coatings made of rare earth metal oxides are effective for forming insulating coatings on electric vehicle contacts, as well as heat-protective coatings for internal combustion engines and jet engines [23–32]. Carbide and sulfide dielectric films are used in microelectronics for the manufacture of high-quality capacitors as well as for transmitting and receiving devices for communication microwave electronic equipment [34; 35]. It was shown in the papers [11; 12; 23–32], that the best way to obtain such coatings is electron beam evaporation in a vacuum with stimulation of a chemical reaction between metal vapor and the residual gas by igniting an auxiliary low-voltage discharge.

It is clear, that the main part of any electron beam technological equipment is an electron gun, which ensures the generation of an electron beam with specified energy and geometric parameters for predetermined conditions of the technological process. Therefore, when one designing the electron beam technological installation, an extremely important engineering aspect is always the coordination of the physical operating conditions of the electron gun with the parameters of the technological process that is being performed [1–10]. It is especially important to ensure an appropriate pressure range in the area of electron beam formation and in the area of technological operation.

In general, it should be noted that today the elaboration of electron guns for technological use is carried out mainly in two directions: improving the designs of traditionally used guns with heated cathodes [1–10] and the development of electron guns, the generation of beams in which is carried out in a fundamentally different way, for example, through field emission [1–10], photoemission [1–10] or the ignition of various types of gas discharges [11–13; 45–47].

HIGH-VOLTAGE GLOW DISCHARGE ELECTRON GUNS AND THE PARTICULARITIES OF THEIR OPERATION IN INDUSTRIAL TECHNOLOGICAL EQUIPMENT

In the papers [11–13; 23–32], it was pointed out that in the physical conditions of low vacuum, on the order of 0.1–10 Pa, the High-Voltage Glow Discharge (HVGD) electron guns usually operate stably and very reliably. The undeniable advantages of such types of electron sources in comparison with traditionally used electron guns with heated cathodes are the following [11–13; 23–32].

1. The ability of HVGD electron guns to operate in a soft and medium vacuum in the environment of various gases, in particular noble and active ones. This makes it easy to coordinate the gun parameters with the required parameters of the technological process. Typically, the pressure in the discharge chamber of the gun lies in the range of 1–10 Pa. For welding and melting electron beam equipment, it leads to a very important technical and economic effect of simplification of technological installations [16; 17], and the technological process of deposition of high-quality ceramic films in the medium of active gas is generally extremely difficult to implement without the use of HVGD electron guns [23–32]. To coordinate the operating parameters of HVGD electron guns and in the area where technological operation is being performed, pressure decoupling is used through an electron beam transport channel with a limited radius. This makes it possible to maintain the required value of pressure in the HVGD combustion area and in the technological chamber with a controlled injection of gas into the electron gun and continuous pumping of the technological chamber [48; 49].

2. The simplicity of the design of HVGD electron guns and the possibility of their assembly and disassembly in order to replace used components, in particular, the HVGD cold cathode [11; 12]. Rough and precise estimates of the operating physical conditions of the cold HVGD cathode and its surface temperature are given in the papers [16; 17].

3. Ease of debugging HVGD electron guns and ensuring their operation as part of technological electron beam installation [11; 12]. The only point related to the complexity of assembling HVGD electron guns is ensuring the alignment of the design parts [21; 22].

4. The relative simplicity of vacuum evacuation technological equipment, since there is no need to ensure the operation of the HVGD electron guns under high vacuum physical conditions [11; 12].

5. Ease of regulation of the electron beam power at a stable value of the accelerating voltage. There are two ways existed for control the power of the electron beam: aerodynamic, through a controlled change of pressure in the gap of HVGD lighting by regulating the gas flow [48; 49], and electrical, through the ignition of an auxiliary discharge and changing the ion concentration in the anode plasma [17–19]. A generalized description of the operating algorithm of the digital current control system of the HVGD electron gun based on the methods of discrete mathematics and the theory of finite state machines is given in [50].

THE STATE OF DEVELOPMENT OF TECHNOLOGICAL EQUIPMENT WITH HIGH-VOLTAGE GLOW DISCHARGE ELECTRON GUNS AND THE CONSIDERED PROBLEM OF SIMULATION OF TRANSPORT CHANNEL

However, despite the technical and economic advantages of HVGD electron guns described above, there are also certain technical difficulties associated with their

industrial application. They are primarily related to the coordination of the physical operating conditions of HVGD guns with the pressure parameters of the technological process [11; 12]. For welding electron guns and guns intended for melting metals and ceramic materials for the purpose of cleaning them, it is very important to separate the HVGD combustion zone from the zone of product processing [16; 17; 36–41], and for the process of deposition ceramic coatings, it is important to ensure the required pressure in the HVGD combustion zone and in the area of metal's vapor interaction with the operation gas [23–32]. Therefore, usually, to ensure stable operation of HVGD electron guns as part of technological equipment, a pressure decoupling is used between the HVGD combustion area and the area of the technological operation. For this purpose, electron beam transport channels with a limited cross-sectional radius are usually used. Then the pressure difference between the HVGD region and technological chamber is ensured through a controlled injection of gas into the electron gun and continuous pumping of the technological chamber. The corresponding block diagram of the electron beam technological installation is given in [48; 49], in the simplified form it is presented at Fig. 1 [48; 49].

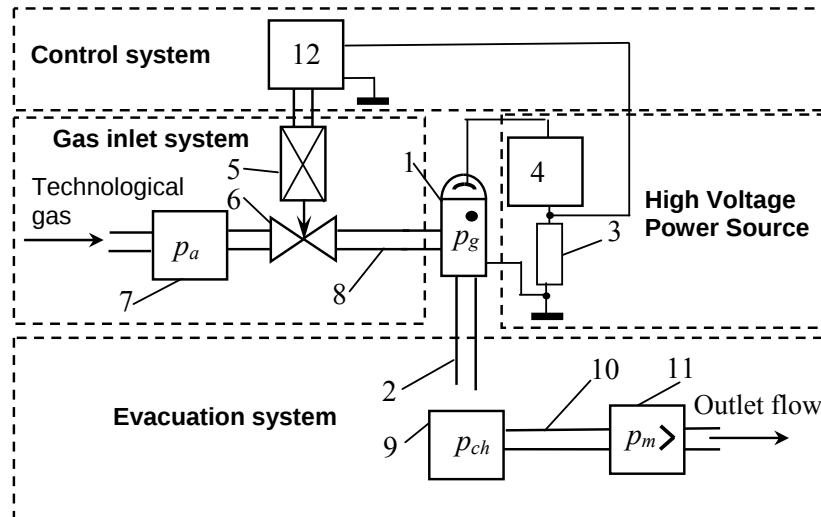


Fig. 1. Block diagram of the pumping and power supply system for an electron beam installation with a HVGD gun: 1 — HVGD electron gun; 2 — channel for guiding an electron beam into the technological chamber; 3 — gun current sensor; 4 — high voltage power supply; 5 — electromagnetic valve for inputting gas into the HVGD electron gun; 9 — technological chamber; 10 — pumping channel; 11 — vacuum pump; 12 — electronic or microcontroller system for automatically changing the gun current; p_a — atmospheric pressure; p_{ch} — pressure in the technological chamber; p_g — pressure in the HVGD electron gun; p_m — minimum pressure in the technological chamber that can be provided by applied pumping means

Typically, transport channels of cylindrical and conical cross-sections are used to guide a short-focus electron beam. Corresponded relations of vacuum technology for calculation the conductivity of such channels relative to the difference of pressure and gas flow are given in the manual books [51–57].

The aim of this paper is to analyze the possibilities of using transport channels with a nonlinear cross-section. An assessment of the vacuum conductivity of a nonlinear channel with power dependence of its radius r on longitudinal coordinate z under the physical conditions of carrying out technological operations for the deposition of ceramic films and coatings has been provided. For electron beam

welding and melting equipment, similar estimates can also be used [1–10; 51–57]. For providing generalized analyze of considered nonlinear function the standard mathematical approaches of function theory and methods for calculating integrals of power functions has been applied [58; 59]. To determine the conductivity of the vacuum channel, well-known equations of vacuum technology were used, and the required length of the channel was determined by numerically solving the complex nonlinear equation using the Steffensen method [60–64].

BASIC ANALYTICAL RELATIONS AND FORMALIZING THE OPTIMIZATION TASK FOR CHOOSING THE GEOMETRY PARAMETERS OF A NON-LINEAR TRANSPORT CHANNEL

In the general case, to determine the pressure distribution along the length of an axisymmetric transportation channel under the condition of the molecular regime of gas flow in it, the basic equation of vacuum technology and the Knudsen equation are used [47–53]. The corresponding system of algebraic and integral equations is written in general form as follows [47–53]:

$$p_{ch} = \frac{p_g U_{ch}}{U_{ch} + S_p}; U_{ch} = \frac{4\bar{v}}{3 \int_0^{l_{ch}} \frac{Hdl}{S_{cr}^2}}; \bar{v} = \sqrt{\frac{8R_0T}{\pi M}}, \quad (1)$$

where U_{ch} is the vacuum conductivity of the channel; H is the perimeter of the transportation channel in cross section; S_{cr} is its area; R_0 is the universal gas constant; T is the gas temperature; M is its molecular weight of gas atoms; p_g and p_{ch} are the pressure in the gun volume and in the technological chamber, according to Fig. 1; U_{ch} is the vacuum conductivity of the transportation channel; S_p is the speed of the pumping system; $\bar{v}$ is the average thermal velocity of movement of gas molecules; l_{ch} is the length of the channel.

At a relatively high pressure in the discharge chamber of the HVG D gun, an intermediate gas flow regime is observed in the electron beam transport channel. In this case, to calculate the channel conductivity, the corresponding correction factor J is introduced [47–53]:

$$J = \frac{1 + 202(R_{in} + R_{out})\bar{p} + 2653((R_{in} + R_{out}))^2}{1 + 236(R_{in} + R_{out})}; U_i = JU_m, \quad (2)$$

Using relations (1), it is possible to determine analytically the vacuum conductivity of where the index m corresponds to the molecular gas flow regime one and the index i for the intermediate one; R_{in} — input radius of beam transporting channel; R_{out} — its output radius correspondently; $\bar{p}$ is the average pressure in the transportation channel; J is a semi-empirical coefficient for listing the conductivity value.

a nonlinear channel for transporting an electron beam with an exponent of $1/\alpha$. The cross section of such a channel, depending on the longitudinal coordinate z , is determined as follows:

$$r(z) = A(z + z_0)^{(1/\alpha)}, \quad (3)$$

where

$$A = \left(\frac{l_{ch}}{R_{out}^\alpha - R_{in}^\alpha} \right)^{(1/\alpha)}, \quad z_0 = \left(\frac{R_{in}}{A} \right)^\alpha = \frac{R_{in}^\alpha (R_{out}^\alpha - R_{in}^\alpha)}{l_{ch}}, \quad (4)$$

or

$$r(z) = \left(\left(\frac{l_{ch}}{R_{out}^\alpha - R_{in}^\alpha} \right) \left(z + \frac{R_{in}^\alpha (R_{out}^\alpha - R_{in}^\alpha)}{l_{ch}} \right) \right)^{(1/\alpha)}. \quad (5)$$

Taking into account the well-known rules of integration of exponential function, solving of integral equation of system (1) giving the following result:

$$U_{ch}(R_{in}, R_{out}, l_{ch}, \alpha) = \begin{cases} \frac{8\bar{v} \left(\frac{\alpha-3}{\alpha} \right)}{3\pi A \left((l_{ch} + z_0)^{\frac{\alpha-3}{\alpha}} - z_0^{\frac{\alpha-3}{\alpha}} \right)}, & \alpha \neq 3; \\ \frac{8\bar{v}}{3\pi A (\ln(l_{ch} + z_0) - \ln(z_0))}, & \alpha = 3, \end{cases}$$

or in the form of arithmetic-logic relation [65]:

$$U_{ch}(R_{in}, R_{out}, l_{ch}, \alpha) = \frac{8\bar{v}}{3\pi} \left((\alpha = 3) \cdot \frac{1}{A (\ln(l_{ch} + z_0) - \ln(z_0))} + \right. \\ \left. + ((\alpha < 3) | (\alpha > 3)) \cdot \frac{\left(\frac{\alpha-3}{\alpha} \right)}{A \left((l_{ch} + z_0)^{\frac{\alpha-3}{\alpha}} - z_0^{\frac{\alpha-3}{\alpha}} \right)} \right). \quad (6)$$

Taking into account relations (3), (4), as well as first relation of set of equations (1), one can obtain the following nonlinear relation for the value l_{ch} relatively to transversal variable z :

$$l_{ch}(z) = \begin{cases} \frac{8\bar{v} \left(\frac{\alpha-3}{\alpha} \right) (p_{ch} - p_g)}{3\pi A J p_{ch} S_p \left((l_{ch} + z_0)^{\frac{\alpha-3}{\alpha}} - z_0^{\frac{\alpha-3}{\alpha}} \right)}, & \alpha \neq 3; \\ \frac{8\bar{v} (p_{ch} - p_g)}{3\pi A J p_{ch} S_p (\ln(l_{ch} + z_0) - \ln(z_0))}, & \alpha = 3, \end{cases}$$

or, in the form of arithmetic-logic relation [65]:

$$l_{ch}(z) = \frac{8\bar{v}(p_{ch} - p_g)}{3\pi p_{ch} S_p} \left((\alpha < 3) \vee (\alpha > 3) \frac{\left(\frac{\alpha-3}{\alpha}\right)}{A \left((l_{ch} + z_0)^{\frac{\alpha-3}{\alpha}} - z_0^{\frac{\alpha-3}{\alpha}} \right)} + \right. \\ \left. + (\alpha = 3) \cdot \frac{1}{A \ln(l_{ch} + z_0) - \ln(z_0)} \right). \quad (7)$$

In further considerations, let us assume that $\alpha > 3$. Therefore, the special case $\alpha = 3$ in arithmetic-logic relation (7) is out of consideration. To solve the nonlinear equation systems (5), (6) with respect to the parameter l_{ch} , let us introduce the corresponding auxiliary variables:

$$a = R_{out}^\alpha; b = R_{in}^\alpha. \quad (8)$$

With this substitution the analytical relations (4), (5) are rewritten as follows:

$$b^2 - b(a + (r(z))^\alpha) - ((r(z))^\alpha a + z l_{ch}) = 0. \quad (9)$$

Clear, that (9) is the quadratic equation relatively to parameter b and for fixed value of a and knowing set of (r, z) coordinate it can be solved analytically.

Let formulate the task of finding nonlinear channel parameters as the task of optimization [61; 62]. Assume, that optimization is provided by four geometry parameters of channel, namely: R_{in} , R_{out} , α and l_{ch} . In such conditions for four basic points $\mathbf{P}_1(r_1, z_1)$, $\mathbf{P}_2(r_2, z_2)$, $\mathbf{P}_3(r_3, z_3)$ and $\mathbf{P}_4(r_4, z_4)$ the relations (8), (9) are rewritten as the following set of nonlinear equations, which can be solved numerically:

$$\begin{cases} R_{in}^{2\alpha} - R_{in}^\alpha (R_{out}^\alpha + (r_1(z_1))^\alpha) - ((R_{out}^\alpha r_1(z_1))^\alpha + z_1 l_{ch}) = 0; \\ R_{in}^{2\alpha} - R_{in}^\alpha (R_{out}^\alpha + (r_2(z_2))^\alpha) - ((R_{out}^\alpha r_2(z_2))^\alpha + z_2 l_{ch}) = 0; \\ R_{in}^{2\alpha} - R_{in}^\alpha (R_{out}^\alpha + (r_3(z_3))^\alpha) - ((R_{out}^\alpha r_3(z_3))^\alpha + z_3 l_{ch}) = 0; \\ R_{in}^{2\alpha} - R_{in}^\alpha (R_{out}^\alpha + (r_4(z_4))^\alpha) - ((R_{out}^\alpha r_4(z_4))^\alpha + z_4 l_{ch}) = 0. \end{cases} \quad (10)$$

The criterium of optimization for the task of beam transporting in nonlinear guiding channel is minimum current losses of electron beam in the case of providing required difference of pressure in the electron gun and in the technological chamber $p_g - p_{ch}$, correspond to Fig. 1 [1–10]. For defining current losses in the transporting channel, the boundary trajectory of electron beam has to be calculated. In such case beam losses on the small elementary range of longitudinal coordinate dz are defined as follows [1–10]:

$$\beta_b(n) = \sqrt{\frac{I_b(n)}{2\pi j_0(n)}}; \quad \frac{I_b(n)}{\pi r_b^2(n)} = j_0 \exp\left(\frac{r_b(n)}{\beta_b(n)}\right)^2; \\ dI_b(n) = \pi \beta_b^2(n) j_0(n) \left\{ \exp\left[-\left(\frac{r_{ch}(n) - \beta(n)}{2}\right)^2\right] - \exp\left[-\left(\frac{r_{ch}(n)}{2}\right)^2\right] \right\}, \quad (11)$$

$$I_b(n+1) = I_b(n) + dI_b(n), \quad r_{ch}(n) = r_{ch}(ndz), \quad r_{ch}(n+1) = r_{ch}((n+1)dz), \quad dz = \left[\frac{l_{ch}}{N_{it}} \right],$$

where I_b — beam current; β_b — beam parameter, which characterized the dissipation of electrons by the velocity; n — current number of iteration, N_{it} — total number of iterations [1–10]. Generally, algorithm of calculation of boundary trajectories of electron beam in the case of its' propagation in the free space with the constant pressure have been described in the papers [66–72]. Also, in this papers an advanced method of interpolation and approximation of boundary trajectories of electron beam, based on root-polynomial function and giving very small error, has been proposed and analyzed. Using such approach, the optimization task in this case is formulated as follows [61; 62]:

$$\begin{pmatrix} R_{in} \\ R_{out} \\ l_{ch} \\ \alpha \end{pmatrix} = \min \left(\sum_{z=0}^{l_{ch}} dI_b(z) \right). \quad (12)$$

Since the channel radius, defined from relations (4), (5), is depend on channel geometry parameter, the coordinates of basic points $\mathbf{P}_1 - \mathbf{P}_4$, as a result of solving set of equations (10), are correspond to criterium (12). Usually, it is enough to define 4 point on the beam boundary trajectory, for which beam radius have the maximal value and choose the channel radius by the simple relation [1–10; 45–48]:

$$r(z) = 3 \beta_b r_b(z). \quad (13)$$

Relation (13), from the point of view of the physics of HVGD [12; 45–47], is explained by the fact that with a large number of collisions of beam electrons with residual gas atoms, the longitudinal distribution of current density $j(r)$ corresponds to the Gaussian law with a very high probability, more than 99% [1–4].

Generally, analyzing beam current losses using relations (11)–(13) is the separate sophisticated problem for future research, which consideration is not the subject of this paper. However, in any case, numerical solving of the set of equations (10) with the known set of basic points $\mathbf{P}_1 - \mathbf{P}_4$ coordinates is a separate complex problem because this set of equations has been formed by using the stiff power function (5) [63; 64]. The corresponding proposed algorithm for solving the set of equations (10), based on the Steffensen method [63; 64], will be described in the next part of the article.

THE NUMERICAL ALGORITHM FOR DEFINING THE GEOMETRY PARAMETERS OF A NON-LINEAR TRANSPORT CHANNEL

Having solved the quadratic equation (9) for the variable b and substituting, instead of the common variables z and r , its values z_1 and r_1 , it is easy to obtain following relation for the function $f_b(a, l_{ch}, \alpha)$:

$$b = f_b(a, l_{ch}, \alpha) = \frac{1}{2}(a + r_1(z_1))^\alpha + \sqrt{a^2 + 2(r_1(z_1))^\alpha + (r_1(z_1))^\alpha - 4(z_1 l_{ch} - a r_1(z_1))^\alpha}. \quad (14)$$

Further, if one rewrites relation (9) with respect to the variable a and substitutes, instead of the common variables z and r , their values z_2 and r_2 , then the corresponding relation for the function $f_a(b, l_{ch}, \alpha)$ will be written as follows:

$$a = f_a(b, l_{ch}, \alpha) = \frac{z_2 l_{ch} + b(r_2(z_2))^\alpha - b^2}{2(r_2(z_2))^\alpha - b}. \quad (15)$$

Similar, from relation (9) the function for calculation the channel length l_{ch} by substitutes, instead of the common variables z and r , their values z_3 and r_3 , is written as follows:

$$l_{ch} = f_l(a, b, \alpha) = \frac{(r_3(z_3))^\alpha (a - b) - b^2 + ab}{z_3}. \quad (16)$$

And finally, the function for defining the exponent α in relations (3), (4), through the known values a , b , l_{ch} , and substitutes, instead of the common variables z and r , their values z_4 and r_4 , is written as follows:

$$\alpha = f_\alpha(a, b, l_{ch}) = \frac{\ln(z_4 l_{ch} - b^2 + ab) - \ln(a + b)}{\ln(r_4(z_4))}. \quad (17)$$

Taking into account relation (14)–(17), the values a, b, l_{ch} , and α , which are satisfied for set of equations (10) for given coordinates of basic points $\mathbf{P}_1 - \mathbf{P}_4$, can be defined numerically using the Steffensen method of solving sets of nonlinear equations [62–64]. Corresponding numerical relations for iterative calculation of these values are written as follows:

$$b_n = \frac{f_b^2(a_{n-1}, l_{ch_{n-1}}, \alpha_{n-1}, r_1, z_1)}{f_b(a_{n-1} + f_a(b_{n-1} + f_b(a_{n-1}, l_{ch_{n-1}}, \alpha_{n-1}, r_1, z_1), l_{ch_{n-1}}, \alpha_{n-1}, r_1, z_1), l_{ch_{n-1}}, \alpha_{n-1}, r_1, z_1) - f_b(a_{n-1}, l_{ch_{n-1}}, \alpha_{n-1}, r_1, z_1)}; \quad (18)$$

$$a_n = \frac{f_a^2(b_n, l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2)}{f_a(b_n + f_b(a_{n-1} + f_a(b_n, l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2), l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2), l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2) - f_a(b_n, l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2)}; \quad (19)$$

$$\alpha_n = \frac{f_\alpha^2(a_n, b_n, l_{ch_{n-1}}, r_3, z_3)}{f_\alpha((a_n + f_a(b_n, l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2)), b_n, l_{ch_{n-1}}, r_3, z_3) - f_\alpha(a_n, b_n, l_{ch_{n-1}}, r_3, z_3)}; \quad (20)$$

$$l_{ch_n} = \frac{f_l^2(a_n, b_n, \alpha_{n-1}, r_4, z_4)}{f_l((a_n + f_a(b_n, l_{ch_{n-1}}, \alpha_{n-1}, r_2, z_2)), b_n, \alpha_{n-1}, r_4, z_4) - f_l(a_n, b_n, \alpha_{n-1}, r_4, z_4)}, \quad (21)$$

where n is number of current iteration.

Iterative calculations using the given relations (14)–(21) have been carried out until the vacuum conductivity of the transport channel reaches a value that

provides the required pressure difference $p_g - p_{ch}$, corresponding to Fig. 1, while the channel conductivity have been calculated from relation (6). Correspondently, iterative calculations have been considered completed if the modulus of the difference in channel conductivities at the previous and current iterations did not exceed $10^{-6} \frac{\text{Pa} \cdot \text{m}^3}{\text{s}}$, namely:

$$\delta = |U_{ch_n} - U_{ch_{n-1}}| < 10^{-6} \frac{\text{Pa} \cdot \text{m}^3}{\text{s}}, \quad (22)$$

where δ is achieved accuracy of calculations on the current iteration n .

The corresponding flow chart of the described iterative algorithm of calculations, which have been carried out using relations (6; 7; 14–22), is presented in Fig. 2.

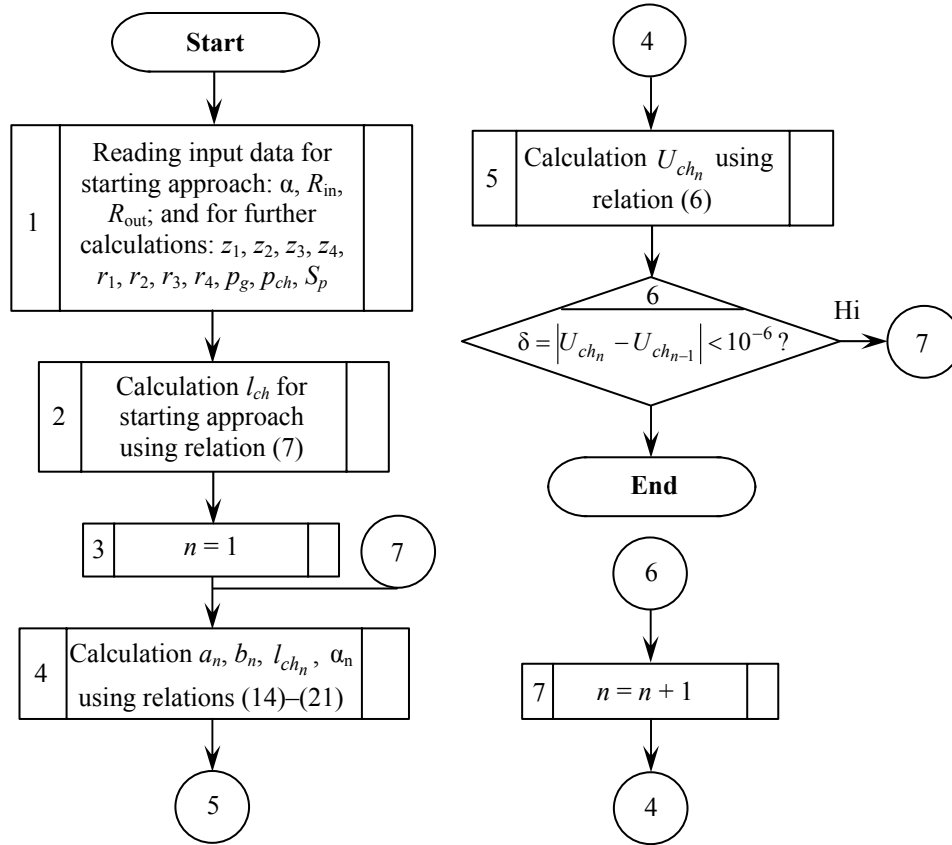


Fig.2. Flow chart of described iterative algorithm for define geometry parameters of nonlinear vacuum channel for transporting electron beam

The testing results of using proposed and described algorithm for solving the task of designing electron-beam vacuum equipment will be presented in the next part of the article. All numerical calculations have been provided using programming means, as well as numerical and graphic libraries of MATLAB software for scientific and technical calculations [73]. Among advanced programming means of MATLAB structure and matrix approach have been implemented [65; 73].

OBTAINED RESULTS OF NUMERICAL EXPERIMENTS AND ITS DISCUSSION

Let's considering and analyzing in this part of article the several task of defining the geometry parameters of nonlinear transporting channel. In all tasks, which will be considered below, on the start iteration such geometry parameters of nonlinear tube have been taken: $R_{in}=0.006$ m, $R_{out}=0.015$ m, $\alpha=3.5$. All calculations have been provided for such parameters of electron-beam equipment, corresponding to Fig. 1: $p_g=5$ Pa, $p_{ch}=0.1$ Pa, $S_p=0.001 \frac{m^3}{s}$. Starting value for the length of channel has been calculated using relation (7). Therefore, only the coordinates of basic points have been changed in considered tests.

Task 1. $z_1=0.01$ m, $z_2=0.02$ m, $z_3=0.03$ m, $z_4=0.04$ m; $r_1=0.017$ m, $r_2=0.02$ m, $r_3=0.021$ m, $r_4=0.022$ m. Obtained solution: $R_{in}=0.0429$ m, $R_{out}=0.0825$ m, $l_{ch}=0.0691$ m, $\alpha=5.0496$. This solution has been obtained after 23 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task is presented at Fig. 3 as straight line.

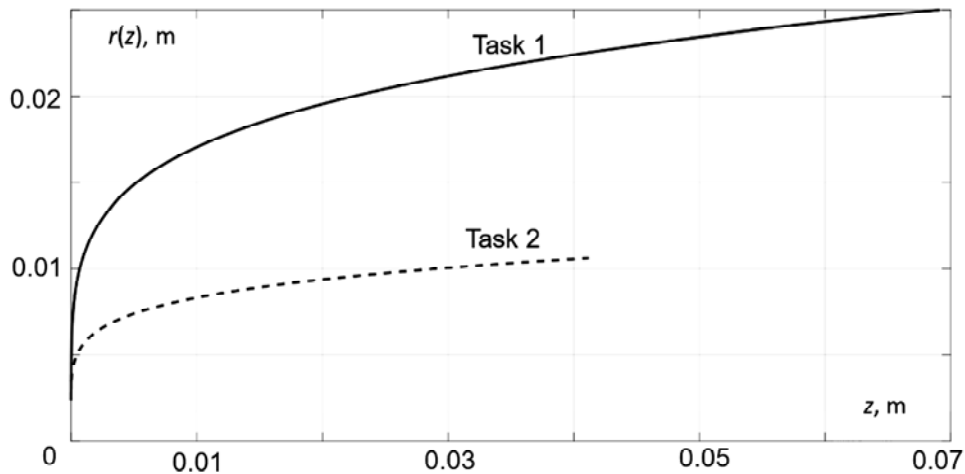


Fig. 3. Graphic dependences $r_{ch}(z)$, obtained using iterative relations (14)–(21) for data sets of Task 1 and Task 2

Task 2. $z_1=0.01$ m, $z_2=0.02$ m, $z_3=0.03$ m, $z_4=0.04$ m; $r_1=0.008$ m, $r_2=0.009$ m, $r_3=0.01$ m, $r_4=0.012$ m. Obtained solution: $R_{in}=0.0701$ m, $R_{out}=0.1128$ m, $l_{ch}=0.0412$ m, $\alpha=5.7743$. This solution has been obtained after 28 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task is presented at Fig. 3 as dash line.

Task 3. $z_1=0.01$ m, $z_2=0.02$ m, $z_3=0.03$ m, $z_4=0.04$ m; $r_1=0.0066$ m, $r_2=0.0085$ m, $r_3=0.01$ m, $r_4=0.011$ m. Obtained solution: $R_{in}=0.0062$ m, $R_{out}=0.0153$ m, $l_{ch}=0.2234$ m, $\alpha=2.7278$. This solution has been obtained after 13 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task for all range of changing of longitudinal coordinate z is presented at Fig. 4, a, and for the start

range of changing z coordinate, correspondently, in Fig. 4, *b*. This solution is reflected in Fig. 4 *a, b*, as straight line.

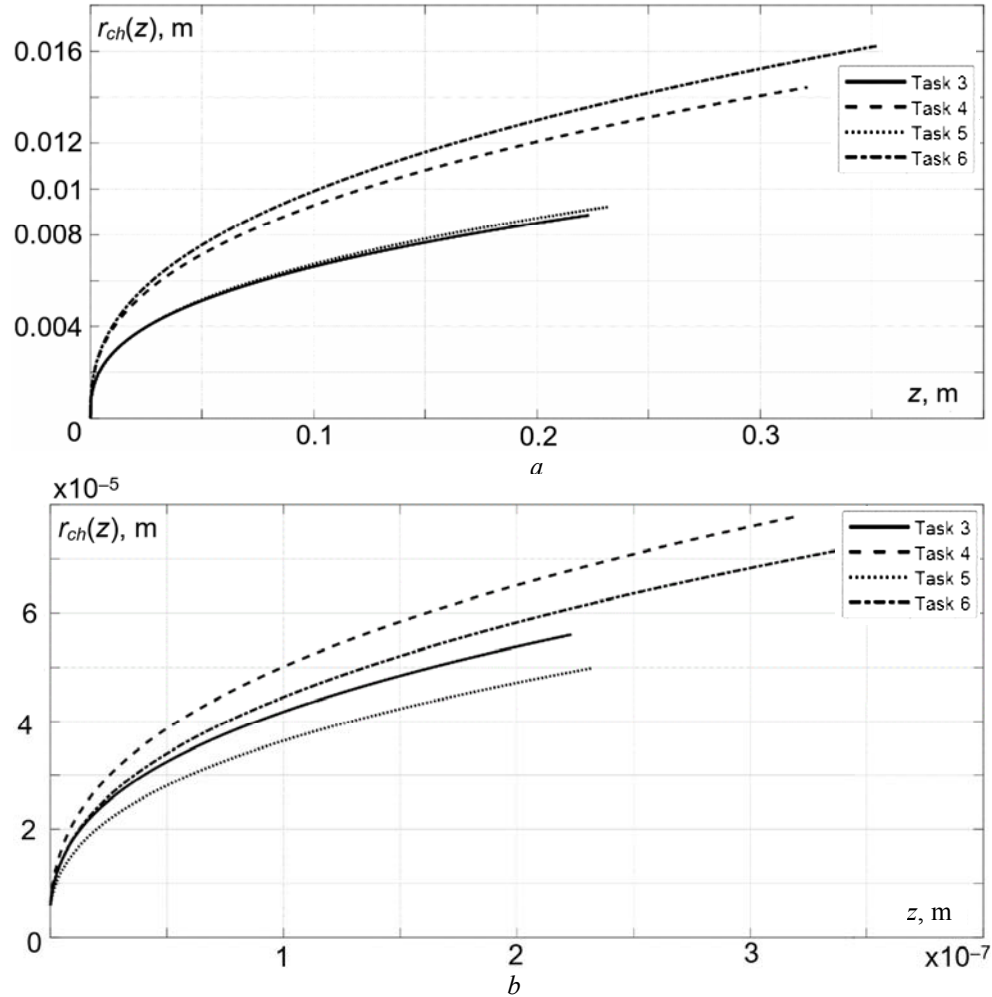


Fig. 4. Graphic dependences $r_{ch}(z)$, obtained using iterative relations (14)–(21) for data sets of Tasks 3, 4, 5 and 6 in the all range (*a*) and in start range (*b*) of changing of longitudinal coordinate z

Task 4. $z_1 = 0.01$ m, $z_2 = 0.02$ m, $z_3 = 0.03$ m, $z_4 = 0.04$ m; $r_1 = 0.0093$ m, $r_2 = 0.0121$ m, $r_3 = 0.0141$ m, $r_4 = 0.0157$ m. Obtained solution: $R_{in} = 0.0059$ m, $R_{out} = 0.0151$ m, $l_{ch} = 0.32211$ m, $\alpha = 2.64456$. This solution has been obtained after 15 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task for all range of changing of longitudinal coordinate z is presented at Fig. 4, *a*, and for the start range of changing z coordinate, correspondently, in Fig. 4, *b*. This solution is reflected in Fig. 4 *a, b*, as dash line.

Task 5. $z_1 = 0.01$ m, $z_2 = 0.02$ m, $z_3 = 0.03$ m, $z_4 = 0.04$ m; $r_1 = 0.0067$ m, $r_2 = 0.0087$ m, $r_3 = 0.0102$ m, $r_4 = 0.0113$ m. Obtained solution: $R_{in} = 0.0061$ m, $R_{out} = 0.0149$ m, $l_{ch} = 0.2323$ m, $\alpha = 2.6456$. This solution has been obtained after 17 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task for all range of

changing of longitudinal coordinate z is also presented at Fig. 4, *a*, and for the start range of changing z coordinate, correspondently, in Fig. 4, *b*. This solution is reflected in Fig. 4 *a, b*, as dot line.

Task 6. $z_1 = 0.01$ m, $z_2 = 0.02$ m, $z_3 = 0.03$ m, $z_4 = 0.04$ m; $r_1 = 0.0099$ m, $r_2 = 0.013$ m, $r_3 = 0.0152$ m, $r_4 = 0.0171$ m. Obtained solution: $R_{in} = 0.0063$ m, $R_{out} = 0.0152$ m, $l_{ch} = 0.352$ m, $\alpha = 2.5546$. This solution has been obtained after 14 iterations. Obtained graphic dependence $r_{ch}(z)$ for this task for all range of changing of longitudinal coordinate z is also presented at Fig. 4, *a*, and for the start range of changing z coordinate, correspondently, in Fig. 4, *b*. This solution is reflected in Fig. 4 *a, b*, as dash-dot line.

From the obtained calculation results and graphical dependencies presented in Fig. 3 and Fig. 4, it is clear that the iterative numerical algorithm based on relations (14)–(21), the flow chart of which is presented in Fig. 2, converges stably for various sets of numerical data for base points $P_1 - P_4$. It should be noted that this is a very important practical result, since the numerical solution of sets of equations containing power and logarithmic functions with a high degree of rigidity cannot always be implemented using standard numerical methods [62–64].

A good proof of the stable convergence of the proposed iterative method is that for close values of the base point data sets, the solutions to the tasks posed are almost identical. This is clearly visible in the obtained solutions for Task 3 and Task 5. Indeed, as can be seen from the graphical dependencies shown in Fig. 4, *a*, the solutions obtained for given sets of points with close coordinates practically coincide.

It should also be noted that all the considered examples are of a practical nature, and the numerical data sets given in problems 1–6 correspond to the actual dimensions of the channels for transporting electron beams in industrial technological equipment. For example, with an input diaphragm radius of 8 mm and a radius of a cylindrical beam transportation channel of 16 mm, a pressure drop

from 5 Pa to 0.1 Pa at a pumping speed of $0.001 \frac{\text{m}^3}{\text{s}}$ is provided with a transportation channel length of 0.28 m.

It is possible in the model task to consider such a transporting channel as a nonlinear one described by the function (4) with parameter $\alpha = 20$. The calculation results for such a model of the beam transport channel give a result of 0.24 m, which is in good agreement with experimental data, taking into account the complexity of the simulation of real vacuum systems [51–57]. To further harmonize theoretical and experimental data, empirical coefficients can be introduced into calculation formulas (14)–(17). In any case, the stable convergence of iterative formulas (18)–(21) is an undeniable advantage of the proposed algorithm for solving important practical problems of modern electron beam technologies.

To carry out further theoretical research in order to solve complex practical and engineering problems of modern industrial electron beam technologies, it is necessary to combine the numerical method described in this article for calculating the geometric parameters of the vacuum channel for transporting an electron beam with the previously proposed modern methods of interpolation and approximation of the boundary trajectories of an electron beam propagating in soft

vacuum [66–72]. In this way, the important practical problem of optimizing beam current losses in the guide channel, described by equations (11)–(13), can be solved.

All research work described in this paper has been provided in the Scientific and Educational Laboratory of Electron Beam Technological Devices of the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnical Institute”.

CONCLUSION

The test numerical experiments carried out in the research work showed that the proposed algorithm for the numerical calculation of the geometric parameters of a nonlinear vacuum channel for transporting a short-focus electron beam, specified by iterative relations (14)–(21) and shown in the form of a flow chart in Fig. 2, converges stably for the pumping speed of the vacuum chamber and the range of pressures in the electron gun and in the technological chamber used in electron beam technological installations. The obtained results of numerical calculations are in good agreement with experimental data. The divergence of calculated and experimental data for test tasks did not exceed 10%. The implementation of the proposed algorithm together with the solution of problems of interpolation and approximation of the boundary trajectory of an electron beam in a single software package will allow, at the initial stage of designing electron beam technological equipment, to estimate the energy losses of the electron beam in the transportation channel and to study complexly the issue of the efficiency of applying of nonlinear beam transportation channels in industrial installations, taking into account the complexity of their production.

Currently, for the manufacture of beam transportation channels with nonlinear geometry, electron beam technologies of three-dimensional metal printing can be applied. In such cases, the manufacturing technology of curvilinear tubes is significantly simplified, and the energy costs as well as the consumption of the material used are also valuably reduced. The accuracy of manufacturing a nonlinear channel using the electron beam three-dimensional printing method is also much higher than when using traditional mechanical processing methods, and the mechanical and thermodynamic properties of the resulting metal are always much better [43; 44].

The results presented in the article may be of great practical interest to experts involved in the development of electron beam equipment and its applying in modern industry. Experts in the field of computational mathematics and numerical methods may be interested in the proposed algorithm for the numerical solution of a stiff set of nonlinear equations (10). The distinguishing feature of this algorithm stably converges for different values of the power function exponent $1/\alpha$.

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Received 23.01.2024

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ЧИСЛОВИЙ АЛГОРИТМ РОЗРАХУНКУ ВАКУУМНОЇ ПРОВІДНОСТІ НЕЛІНІЙНОГО КАНАЛУ ТРАНСПОРТУВАННЯ КОРОТКОФОКУСНОГО ЕЛЕКТРОННОГО ПУЧКА У ТЕХНОЛОГІЧНОМУ ОБЛАДНАННІ / І.В. Мельник, А.В. Починок, М.Ю. Скрипка

Анотація. На основі розв’язування рівнянь вакуумної техніки запропоновано і досліджено ітераційний алгоритм розрахунку вакуумної провідності та геометричних параметрів криволінійного каналу транспортування короткофокусного електронного пучка, для якого залежність радіуса каналу від позовжньої координати описують степеневою функцією. Запропонований алгоритм заснований на числовому розв’язуванні системи нелінійних рівнянь методом Стеффенсена. Наведені результати тестових розрахунків підтверджують стійку збіжність запропонованого алгоритму для реальних значень тиску та швидкості відкачування у технологічному обладнанні. Такі криволінійні канали транспортування можуть бути використані в електронно-променевому обладнанні на основі гармат високовольтного тліючого розряду, призначеному для зварювання, плавлення металів та для нанесення тонких плівок. Критерієм оптимальності геометрії нелінійного каналу є мінімальні втрати потужності електронного пучка під час його транспортування за умови забезпечення необхідного перепаду тиску між розрядною та технологічною камерами.

Ключові слова: електронний пучок, електронно-променеві технології, транспортування електронного пучка, нелінійний канал транспортування електронного пучка, вакуумна провідність каналу транспортування, електронна гармата високовольтного тліючого розряду, рівняння вакуумної техніки, система нелінійних рівнянь, метод Стеффенсена.