

STRATEGY FOR ENSURING ASYMPTOTIC CONVERGENCE OF THE PROCESS OF NON-LINEAR ESTIMATION OF DYNAMIC OBJECT PARAMETERS

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Abstract. The article considers a step-by-step strategy of sequential use and adjustment of a parallel model to an object of identical structure with orthogonal operators, a series-parallel model to an object with the connection of operators of a certain type for orthogonal approximation in order to obtain asymptotically unbiased estimates of coefficients of a structurally identical to a dynamic object of a mathematical model under conditions of noise of measurements of the initial variable of the identification object and non-convexity of the proximity functional of the initial variables of the object and the model in a space of coefficients of the object's mathematical model. Structural diagrams of each stage of identification are given using refined parameters and the structure of the model of object. This algorithm was implemented to identify the parameters of the mathematical model of aircraft, provided that the sample of experiment data is limited and there is of the initial a significant range of deviations of state variables from the basic mode.

Keywords: non-linear estimation, identification, convergence of estimation algorithms, optimization.

INTRODUCTION

An important place in the problems of non-linear programming (identification [1–8]) is occupied by the form of a proximity functional of initial variables of an object and a model, whose physical parameters are optimized at the extremum (usually the minimum) of the variable functional. This functional is simultaneously a function of physical parameters of the model being optimized. If the “input (x) – output (y)” mapping of the object can be represented by the following differential equation:

$$\begin{aligned} a_0 \frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_{n-1} \frac{dy}{dt} + a_n y(t) = \\ = b_0 \frac{d^m x}{dt^m} + b_1 \frac{d^{m-1} x}{dt^{m-1}} + \dots + b_{m-1} \frac{dx}{dt} + b_m x(t), \end{aligned} \quad (1)$$

where $a_i, i = \overline{0, n}; b_j, j = \overline{0, m}$ are parameters to be determined using entries $y(t_k), x(t_k), k = \overline{1, N}$, considering that $y(t_k)$ is measured with uncorrelated noises $\zeta(t_k)$; then desired coefficients a_i of the model (1) enter the equation of error $\varepsilon(t_k)$ between $y(t_k)$ and output variable $y_M(t_k)$ non-linearly, since they are in the denominator of the model operator $W_M(p)$:

$$\varepsilon(t) = y(t) - W_M(p)x(t),$$

where $W_M(p) = \frac{b(p)}{a(p)}$, $p = \frac{d}{dt}$, $b(p) = b_0p^m + b_1p^{m-1} + \dots + b_m$, $a(p) = a_0p^n + a_1p^{n-1} + \dots + a_n$.

Accordingly, parameters a_i non-linearly enter mean square $\overline{\varepsilon^2}$ of error $\varepsilon(t_k)$ (both for the functional of $y(t_k)$, $y_M(t_k)$ and the functions of parameters a_i, b_j), violating the elliptic form of dependence $\overline{\varepsilon^2}$ on deviations Δa_i of estimates a_i from their optimal values a_i^* :

$$(a_i^*, b_j^*) = \arg \min_{a_i, b_j} \overline{\varepsilon^2}. \quad (2)$$

Violation of ellipticity and strict convexity of function $\overline{\varepsilon^2}(a_i, b_j)$ (the “ravine” effect) significantly complicates the recurrent process of convergence of parameters a_i, b_j with their optimal values a_i^*, b_j^* that satisfy the condition (2).

Furthermore, the form of function $\overline{\varepsilon^2}(a_i, b_j)$ significantly depends on the bandwidth of input signal $x(t)$, because $\overline{\varepsilon^2}$ is simultaneously a functional of $y(t)$, $y_M(t)$, which, in turn, depend on input stimulus $x(t)$. For the given $x(t)$ and the structure of the model (1), the key to successful optimization of the relaxation process for convergence of the model (1) parameter estimates with their optimal values (2) can be a strategy of using a set of different models and a series of their connection and identification, which ensures convexity and ellipticity $\overline{\varepsilon^2}(a_i, b_j)$.

FORMULATION OF THE PROBLEM

While using an object-parallel:

- model with a structure identical to the object (1);
- model with a series-parallel connection of operators $\frac{a(p)}{c(p)}$ and $\frac{b(p)}{c(p)}$ to

the object (where $c(p)$ is a filter of the degree p exceeding the degree n of polynomial $a(p)$), which ensures the correctness of differentiation operation $y(t)$;

- as well as model with orthogonal parameters connected in parallel to the object; it is necessary to organize the sequence of their connection and adjustment in such a way as to ensure strict convexity and ellipticity of proximity functions of the object and the model being optimized, and thus, the convergence of the relaxation process for identification of the model (1) coefficients under the condition (2).

Strategy for implementing the condition of guaranteed convergence of model (1) parameters with their optimal values (2)

The strategy consists of four steps, where Steps 2, 3, 4 are to be repeated until condition (2) is met.

Step 1. Identification (Fig. 1) of weight coefficients β_i of operators $W_i(p)$ of the model (3).

An equation of the model parallel to the object:

$$y_M(t) = \sum_{i=1}^n \beta_i \{W_i(p) \cdot x(t)\} = \sum_{i=1}^n \beta_i \varphi_i(x(t)), \quad (3)$$

where $\varphi_i(x) = W_i(p)x(t)$ are linearly independent functions of time t .

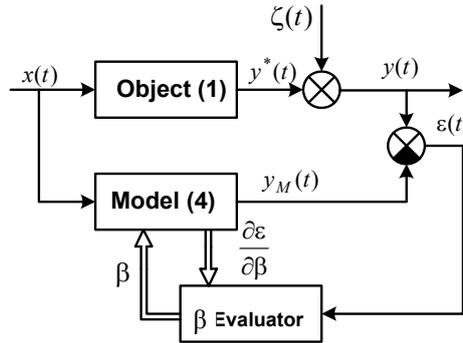


Fig. 1. Approximation of the “input-output” mapping (1) of the object in the model (3)

Operators $W_i(p)$ transform input stimulus $x(t)$ into a system of linearly independent functions $\varphi_i(x(t))$. If $x(t)$ is close to “white noise”, then, using, for instance, Lager functions as these operators

$$W_i(p) = \frac{(p - \gamma)^i}{(p + \gamma)^i}, \quad (4)$$

(where γ is the operator parameter), we obtain a system of mutually orthogonal functions $\varphi_i(x(t))$. Linear independence, and even more so, orthogonality $\varphi_i(x(t))$ guarantee strict convexity and ellipticity of mean square $\overline{\varepsilon^2}$ of error $\varepsilon(t_k)$ [1], [2]. In the “off-line” mode, the determination of the model (3) coefficients β_i can be one-step, if we have a data sample $x(t_k), y(t_k), k = \overline{1, M}$. Then estimates $\hat{\beta}_i$ of coefficients β_i are determined by the least squares method (LSM) under the condition

$$\hat{\beta} = \operatorname{argmin}_{\beta} \frac{1}{M} \sum_{k=1}^M \varepsilon^2(k),$$

where $\varepsilon(k) = y(k) - \sum_{i=1}^n \beta_i \varphi_i(x(t))$.

Meaning, $\hat{\beta} = (\varphi^T \varphi)^{-1} \varphi^T Y = (\varphi^T \varphi)^{-1} \varphi^T (Y^* + \zeta)$.

LSM estimation $\hat{\beta}$ of the vector of coefficients $\beta_i, i = \overline{1, n}$ will be unbiased, since noise $\zeta(t)$ is uncorrelated with $\varphi_i(t)$, and if $\zeta(t)$ is a Gaussian “white noise”, then estimate $\hat{\beta}$ will have minimal variance. If $x(t)$ is a Gaussian “white

noise”, and operators $W_i(p)$ are type (4), then matrix $\varphi^T \varphi$ will be diagonal, and each component $\hat{\beta}_i$ of vector $\hat{\beta}$ is determined independently:

$$\hat{\beta}_i = \frac{\sum_{k=1}^M \varphi_i(k) y(k)}{\sum_{k=1}^M \varphi_i^2(k)}.$$

In the “on-line” mode, the recurrent process of adjusting coefficients β_i of the model (3) is done through the gradient procedure:

$$\frac{d\hat{\beta}_i}{dt} = \lambda_i \varepsilon(t) \varphi_i(t).$$

If λ_i is limited, the process of approximating estimate $\hat{\beta}_i(t)$ to the optimal stationary value is exponential, which achieves an exponentially weighted averaging of the current value $\varepsilon^2(t)$.

Resulting from operation of the system (Fig. 1) in the first step, with the limited dimensionality n of the model (3), we have a close approximation of mapping $x(t)$ onto $y^*(t)$ in the model (3) (non-parametric identification). In this case, the structures of the desired mapping (1) and the model (3) are different, but y_M no longer contains noise $\zeta(t)$.

Step 2. Approximation of mapping $x(t)$ onto $\hat{y}_M(t)$ that uses the series-parallel model (Fig. 2) of the equation (1).

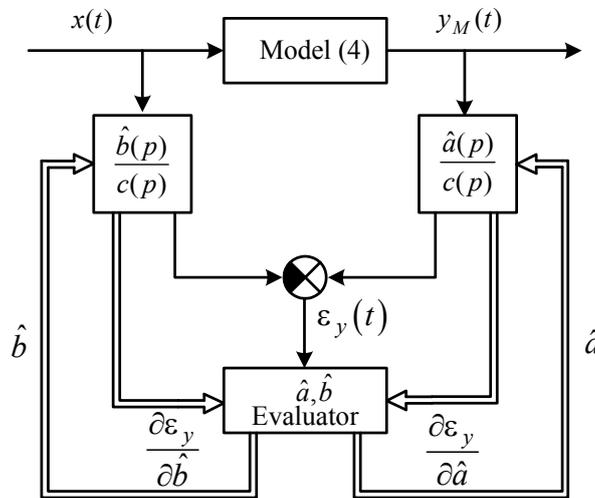


Fig. 2. Approximation of mapping $x(t)$ onto $\hat{y}_M^*(t)$ in the model (1)

Total error equation:

$$\varepsilon_y(t) = \frac{\hat{a}(p)}{c(p)} y_M(t) - \frac{\hat{b}(p)}{c(p)} x(t), \tag{5}$$

where $c(p)$ is a filter polynomial with degree exceeding polynomial degree $a(p)$; polynomial structures $\hat{a}(p)$ and $\hat{b}(p)$ are identical to the polynomial structures $a(p)$ and $b(p)$ of the object model (1). Then, depending on the “on-

line” or “off-line” modes of minimizing the mean square of total error $\varepsilon_y(t)$, by means of the adaptive circuit, the least squares method, or the gradient method, respectively, the optimal values of coefficients $\hat{a}_i, i = \overline{1, n}, \hat{b}_j, j = \overline{1, m}$ are calculated. Absence of “noise” in signal $\hat{y}_M(t)$, and linearity of dependence $\varepsilon_y(t)$ on coefficients \hat{a}_i, \hat{b}_j being adjusted guarantee that their estimates are obtained. However, their values are not yet true values a_i, b_j of the object model (1). This is due to the proximity of mapping $y^*(t)$ in the model (3).

Step 3. Approximation of mapping $x(t)$ onto $y^*(t)$ in the model, in the form of a composition of the series connection of model $\frac{\hat{b}(p)}{\hat{a}(p)}$ obtained in the previous step, and the orthogonal model (3) (Fig. 3).

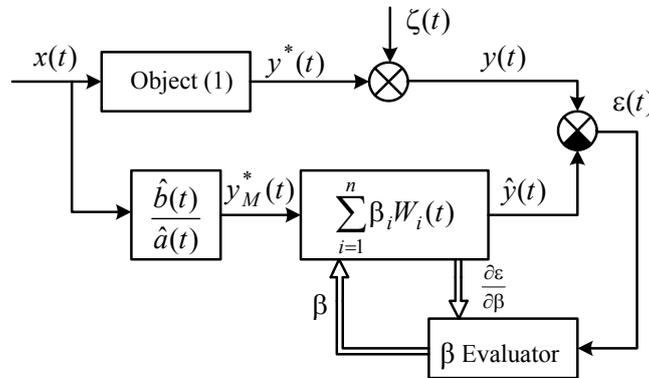


Fig. 3. Structure of the identification system in Step 3

In this step, an inaccuracy of mapping $x(t)$ onto $y^*(t)$ in the model $\frac{\hat{b}(p)}{\hat{a}(p)}$ is compensated by adjusting coefficients β_i of model $\sum_{i=1}^n \beta_i W_i(p)$, which is turned on in series with model $\frac{\hat{b}(p)}{\hat{a}(p)}$ (Fig. 3). The adjustment process β_i is similar to the process in the Step 1 (as per LSM in “off-line” mode, or gradient method in “on-line” mode). Now, however, model $\sum_{i=1}^n \beta_i W_i(p)$ should map not the mapping $x(t)$ onto $y^*(t)$, but only mapping $y_M^*(t)$ onto $y^*(t)$, which is much simpler, since $y_M^*(t)$ is already more or less close to $y^*(t)$.

Step 4 repeats Step 2 (Fig. 2), but for mapping $x(t)$ onto $y^*(t)$ clarified in the model $\sum_{i=1}^n \beta_i W_i(p)$. Theoretically, if in Step 4 n of the model $\sum_{i=1}^n \beta_i W_i(p)$ is unlimited, we will already get estimates \hat{a}_i, \hat{b}_j of parameters a_i, b_j of the identification object’s accurate model (1), which will be unbiased by noise $\zeta(t)$.

In practice, if n is limited, Steps 2–4 are repeated, and $\hat{y}(t)$ gradually approaches $y^*(t)$ of the object, coefficients β_i of the model (3) approach zero, except for β_0 , which approaches one with the unit operator $W_0(p)=1$, and coefficient estimates \hat{a}_i, \hat{b}_j approach true values a_i, b_j of the model (1).

A flow chart of the strategy of using three types of models is presented in Fig. 4.

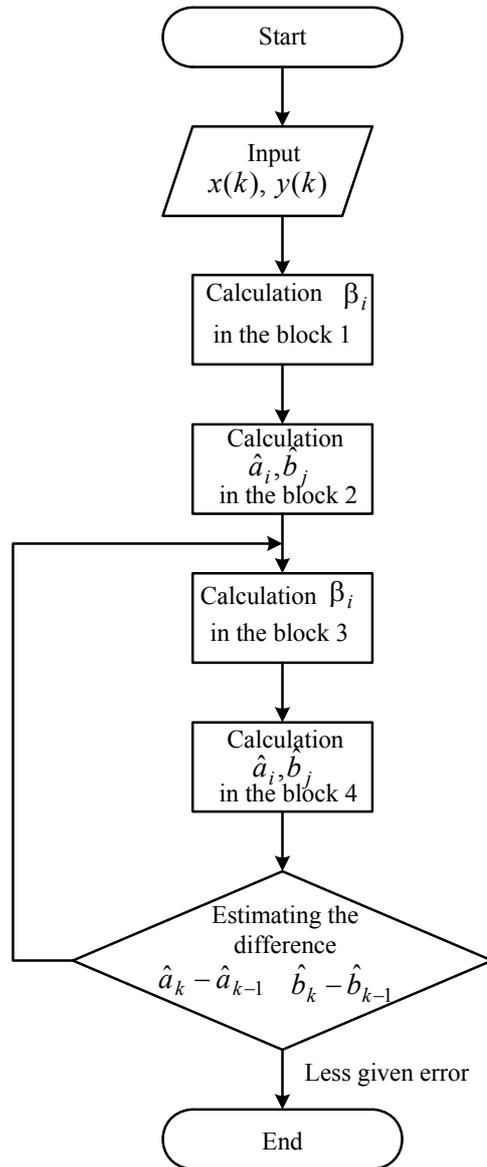


Fig. 4. Flow chart of the identification algorithm for model (1) parameters

increase unacceptably. Desire to reduce impact of non-linearity on a linearized aircraft model leads to an increase in the “noise-to-signal” ratio and, as a result, an increase in the random component of the error in estimating the parameters of the mathematical model (MM) of the aircraft. This range increase leads to a shift in the linear MM parameters’ estimates due to the influence of non-linearity of the

We will consider the feasibility of using the proposed strategy on the example of identifying coefficients of the transfer function of an aircraft in longitudinal short-periodic movement [4]. The problem of identification lies in a concept of the transfer function being valid only under conditions of linearity and stationarity of the mapping of the control stimulus (deviation $\Delta\delta_h(t)$ of the altitude control) in the deviation of attack angle $\alpha(t)$ of the angle between the longitudinal axis of the aircraft and the direction of the oncoming air flow.

Short-periodic movement means movement in a short time interval during which factors not taken into account in model (1) hardly change. These include non-linearity, non-stationarity, speed changes, height, weight, and dimensions of the aircraft, etc. Therefore, the mathematical model of the aircraft movement will be more accurate, the smaller the time and deviation of the variables from a certain basic balancing mode are. If the time and range of deviations of the aircraft state variables from the basic mode tend to zero, the model (1) tends to being perfectly accurate. But in reality, the measurements of the aircraft state variables give not only accurate, but a random component of noise. Then, when the range of the variable deviations decreases, the “noise-to-signal” ratio increases, and, in a limited time interval, the variance of estimates of aircraft parameters will increase unacceptably.

aircraft characteristics. If it is impossible to fulfill other conditions (a large sample of experimental data and deviation range of the state variables from the basic mode), the condition of ellipticity and strict quadraticity of the error functional between $y(t)$ and $y_M(t)$, as a function of the optimized parameters of the aircraft MM, allow to successfully solve the problem of identifying parameters \hat{a}_i and \hat{b}_j of the MM on the level of their proximity to real physical values a_i and b_j of the accurate MM (1).

Let's suppose for small deviations from the basic longitudinal horizontal movement the aircraft MM looks as follows:

$$\frac{d^2 y^*}{dt^2} + a_1 \frac{dy^*}{dt} + a_0 y^*(t) = bx(t),$$

where $y^*(t)$ is the deviation of attack angle $\alpha(t)$, and $x(t)$ is the altitude control step deviation δ_h . Desired aerodynamic coefficients are $a_1 = 1 \text{ c}$, $a_0 = 3$, $b = 0,5$.

Reaction $y^*(t)$ to a single step stimulus $x(t)$ is shown in Fig. 5.

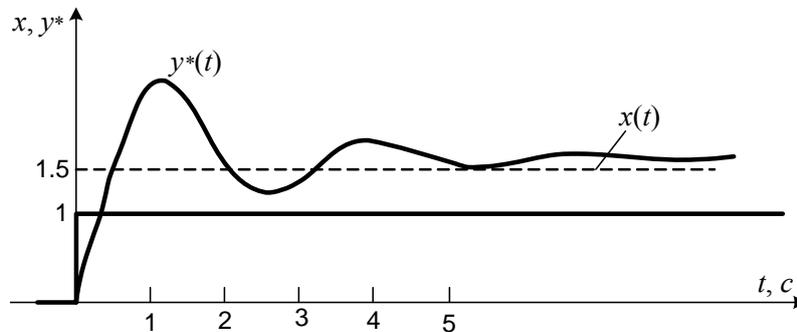


Fig. 5. Reaction $y^*(t)$ to step impact and $x(t)$

The test corresponds to real conditions of aircraft identification: observation time $T = 5 \text{ s}$, discretion step Δt in time was 0.02 s . Meaning, number M of discrete values t_k was 100. Measurements of attack angle $y(t)$ in 100 discrete-times t_k consist of the exact value $y^*(t_k)$ (Fig. 5), and adaptive noise in the form of Gaussian “white noise”. Input stimulus $x(t)$ is measured accurately.

The computer modeling was carried out for various noise $y(t)$ to signal $y^*(t)$ ratios, therefore, due to a limited data sample and the presence of random noise, estimates $\hat{a}_0, \hat{a}_1, \hat{b}$ are random (Table). The estimates were obtained using the series-parallel model (Fig. 2, Step 2) and the three models strategy (Figs. 1–3, Steps 1–4). The model (3) uses three type (4) operators, while the model (5), as a filter $C(p)$, uses operator

$$W_f(p) = (p^3 + 5p^2 + 6p + 7)^{-1},$$

which significantly smooths noise components $\zeta(t)$.

Table shows estimates $\hat{a}_0, \hat{a}_1, \hat{b}$ obtained through a one-step algorithm that uses only the series-parallel model (Step 2), and a multi-step (Steps 1–4) algorithm that uses the proposed strategy (Fig. 4).

The estimates, obtained by the one-step and multi-step algorithms

| Noise/signal | \hat{a}_0 | | \hat{a}_1 | | \hat{b} | |
|--------------|-------------|-----------|-------------|-----------|-----------|-----------|
| | Step 2 | Steps 1–4 | Step 2 | Steps 1–4 | Step 2 | Steps 1–4 |
| 0 | 3 | 3 | 1 | 1 | 0.5 | 0.5 |
| 0.5 | 3.15 | 3.1 | 1.01 | 1.1 | 0.517 | 0.516 |
| 1 | 2.69 | 3.15 | 0.71 | 1.14 | 0.6 | 0.52 |
| 2 | 1.56 | 2.82 | 0.16 | 0.96 | 0.49 | 0.48 |
| 3 | 1.39 | 3.22 | 0.02 | 0.95 | 0.53 | 0.58 |

Therefore, despite a random component of estimates $\hat{a}_0, \hat{a}_1, \hat{b}$, associated with a limited data sample and a significant noise $\zeta(t_k)$ to signal $y^*(t_k)$ ratio, in the dependences \hat{a}_1 and \hat{a}_0 on the noise-to-signal ratio, we can observe a pattern that has a statistically significant value, i.e. a significant decrease in estimates \hat{a}_1 and \hat{a}_0 with an increase in the noise level $\zeta(t_k)$ (Fig. 6).

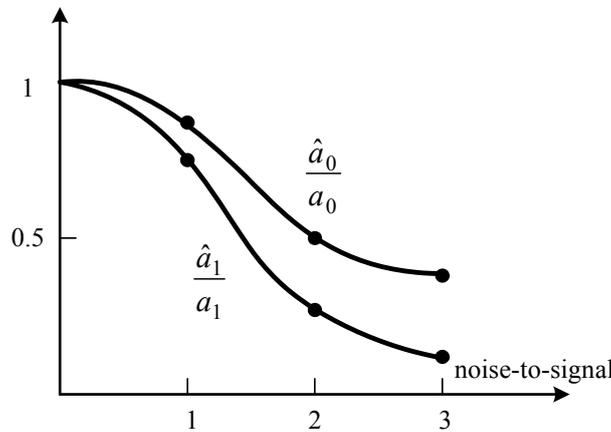


Fig. 6. Dependence $\frac{\hat{a}_1}{a_1}$ and $\frac{\hat{a}_0}{a_0}$ on the noise level $\zeta(t)$ in signal $y(t)$

To explain the effect of lower estimates \hat{a}_i and \hat{a}_0 , we will consider a structural diagram with a series-parallel model (Step 2), where an object is connected instead of a model (Fig. 7).

The LSM evaluator determines estimates $\hat{a}_0, \hat{a}_1, \hat{b}$ of parameters a_0, a_1, b under the condition of the minimum mean square $\bar{\varepsilon}_y^2$ of error $\varepsilon_y(t_k)$ [5; 6], i.e., under the following conditions:

$$\frac{1}{N} \sum_{k=1}^N \varepsilon_y(t_k) \frac{\partial \varepsilon_y(t_k)}{\partial \hat{a}_0} \equiv 0, \tag{6}$$

$$\frac{1}{N} \sum_{k=1}^N \varepsilon_y(t_k) \frac{\partial \varepsilon_y(t_k)}{\partial \hat{a}_1} \equiv 0, \tag{7}$$

$$\frac{1}{N} \sum_{k=1}^N \varepsilon_y(t_k) \frac{\partial \varepsilon_y(t_k)}{\partial \hat{b}} \equiv 0, \tag{8}$$

Total error:

$$\varepsilon_y(t_k) = \hat{a}_0 \hat{y}(t_k) + \hat{a}_1 \frac{d\hat{y}(t_k)}{dt} + \frac{d^2 \hat{y}(t_k)}{dt^2} - \hat{b} \hat{x}(t_k). \tag{9}$$

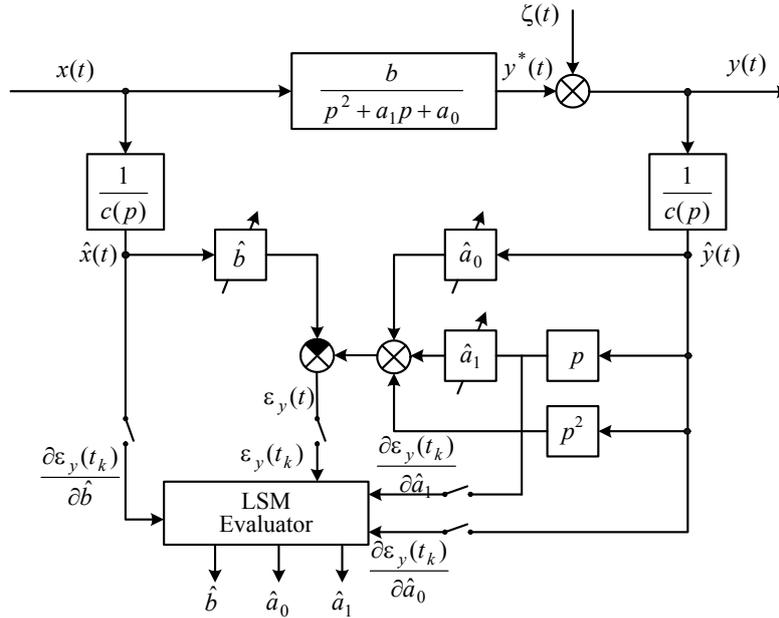


Fig. 7. Evaluation of object parameters using the series-parallel model

Error sensitivity functions for the relevant parameters:

$$\frac{\partial \varepsilon_y(t_k)}{\partial \hat{b}} = \hat{x}(t_k),$$

$$\frac{\partial \varepsilon_y(t_k)}{\partial \hat{a}_0} = \hat{y}(t_k) = \hat{y}^*(t_k) + \hat{\zeta}(t_k),$$

$$\frac{\partial \varepsilon_y(t_k)}{\partial \hat{a}_1} = \frac{d\hat{y}(t_k)}{dt} = \frac{d\hat{y}^*(t_k)}{dt} + \frac{d\hat{\zeta}(t_k)}{dt}.$$

In condition (7), considering expressions (8) and (9), there is no square of noise $\zeta(t)$. If (9) is weakly correlated with the inaccuracy of determining coefficients \hat{a}_0 and \hat{a}_1 in (9), estimate \hat{b} of parameter b is almost unbiased (see Table, Step 2).

It is different for estimates \hat{a}_0 and \hat{a}_1 . In equations (6), (7), there is a square of noise $\zeta(t)$ or its derivative. This leads to a bias in estimates \hat{a}_0 and \hat{a}_1 proportional to the noise level $\zeta(t)$. This is the main drawback of the series-parallel model (Fig. 7), which is eliminated by the proposed strategy (see Table, Steps 2–4).

CONCLUSION

In order to guarantee unbiased physical parameter estimates for mathematical models of dynamic objects in real conditions of limited data samples and dynamic ranges of state variables of an object, an effective strategy consists of a step-by-step use of a parallel model with orthogonal operators, a series-parallel model for approximating the orthogonal model, and a subsequent use of these models for a more accurate “input-output” mapping of the object and, consequently, an unbiased estimation of desired dynamic object parameters.

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СТРАТЕГІЯ ЗАБЕЗПЕЧЕННЯ АСИМПТОТИЧНОЇ ЗБІЖНОСТІ ПРОЦЕСУ НЕЛІНІЙНОГО ОЦІНЮВАННЯ ПАРАМЕТРІВ ДИНАМІЧНИХ ОБ'ЄКТІВ/
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Анотація. Розглянуто покрокову стратегію послідовного використання і налаштування паралельної до об'єкта моделі ідентичної структури з ортогональними операторами, послідовно-паралельної моделі до об'єкта з підключенням операторів певного типу для ортогональної апроксимації з метою отримання асимптотично незміщених оцінок коефіцієнтів структурно ідентичної до динамічного об'єкта математичної моделі в умовах зашумленості вимірів вихідної змінної об'єкта ідентифікації і невивуклості функціонала близькості вихідних змінних об'єкта і моделі в просторі коефіцієнтів математичної моделі об'єкта. Наведено структурні схеми кожного етапу ідентифікації з використанням уточнених параметрів і структури моделі об'єкта. Алгоритм реалізовано для ідентифікації параметрів математичної моделі літальних апаратів за умови обмеженості вибірки даних експерименту і значного діапазону відхилення змінних стану від базового режиму.

Ключові слова: нелінійне оцінювання, ідентифікація, збіжність алгоритмів оцінювання, оптимізація.