

**AUTOMATED CONTROL OF DYNAMIC SYSTEMS  
FOR ENSURING UKRAINE’S SECURITY USING COGNITIVE  
MAP IMPULSE PROCESS MODELS.  
PART 1. DEMOGRAPHIC SECURITY**

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**Abstract.** The paper provides a cognitive map (CM) of demographic security and a dynamic model of CM impulse processes described as a difference equations system (Robert’s equations). The external control vector for the CM impulse process is implemented by means of varying the CM nodes’ coordinates. A closed-loop control system for the CM impulse process is proposed. It includes a multivariate discrete controller designed based on an automated control theory method, which generates the chosen control actions. We solve a discrete controller design problem for automated control of dynamic processes to ensure demographic security. The controller suppresses external and internal disturbances during CM impulse processes control based on the invariant ellipsoids method. The paper presents an algorithm for CM weights identification based on the recurrent least squares method. We present the results of a qualitative research study on dynamic processes related to demographic security in Ukraine under various disturbances during martial law.

**Keywords:** cognitive map, demographic security, invariant ellipsoid, linear matrix inequalities, impulse process.

**INTRODUCTION**

To study the dynamic processes for system ensuring of Ukraine’s demographic security we use cognitive modelling, which is one of the most relevant areas of scientific and practical research of complex systems of different nature now. Cognitive modeling is based on the notion of a cognitive map (CM), which is a weighted directed graph, its nodes reflect coordinates (factors, concepts) of the complex system and weighted edges (arcs) of the graph describe interrelations between CM nodes. When disturbances affect CM nodes, we can observe impulse transitional process, its dynamics is described by the difference equation [1]:

$$\Delta y_i(k+1) = \sum_{j=1}^n a_{ij} \Delta y_j(k), \quad (1)$$

where  $\Delta y_i(k) = y_i(k) - y_i(k-1), i = 1, 2, \dots, n$ ,  $a_{ij}$  — weight of an edge connecting the  $j$ -th node and the  $i$ -th one. Equation (1) describes the free motion of the  $i$ -th

node of CM without external control impact. We can write this equation in vector-matrix form:

$$\Delta\bar{Y}(k+1) = A\Delta\bar{Y}(k), \quad (2)$$

where  $\Delta\bar{Y}(k) = \bar{Y}(k) - \bar{Y}(k-1)$ ,  $A$  is a weighted adjacency matrix of the CM of size  $n \times n$ .

In order to implement control of the CM impulse process (2) based on modern control theory it is necessary to be able to physically change some coordinates of CM nodes as control actions. Then we can describe the forced motion of the CM impulse process under external control as the difference equation:

$$\Delta\bar{Y}(k+1) = A\Delta\bar{Y}(k) + B\Delta\bar{U}(k), \quad (3)$$

where  $\Delta\bar{U}(k) = \bar{U}(k) - \bar{U}(k-1)$  — vector of controls increments with size  $m \leq n$ . The operator fills the control matrix  $B(n \times m)$  and in its simplest form uses ones and zeros.

If the CM has unmeasurable coordinates, they can be included into equation (3) as disturbances. In such a case the impulse process (3) will be written as

$$\Delta\bar{Y}(k+1) = A\Delta\bar{Y}(k) + B\Delta\bar{U}(k) + \Psi\Delta\bar{\xi}(k), \quad (4)$$

where  $\Delta\bar{\xi}(k) = \bar{\xi}(k) - \bar{\xi}(k-1)$  — vector of unmeasurable coordinates (disturbances).

## **PROBLEM STATEMENT**

The first problem is to create a controlled dynamic model of CM impulse process describing multivariate demographic process in Ukraine. The second problem is to research and develop the system for suppressing constrained internal and external disturbances by means of control of the demographic security CM impulse process during martial law. The third problem is to implement an adaptive CM impulse process control under unknown or unmeasurable coefficients of the adjacency matrix  $A$ ; this control should combine procedures of the matrix  $A$  elements' estimation during the transient process and usage of these estimates of the matrix  $\hat{A}$  for a control vector design. The fourth problem is to perform a simulation of the designed closed-loop control system and to research dynamic processes' quality with respect to ensuring demographic security of Ukraine under different disturbances during martial law.

## **CREATION OF A DEMOGRAPHIC SECURITY COGNITIVE MAP**

Fig. 1 represents the schema of the CM of demographic security of Ukraine, developed based on cause and effect relations during martial law. The CM nodes have the following meaning:

0 — state support of families with children; 1 — average salary of a worker; 2 — consumer price index; 3 — export volume; 4 — import volume; 5 — population in Ukraine; 6 — real GDP of Ukraine; 7 — inflation rate; 8 — migration out of Ukraine; 9 — birth rate; 10 — unemployment rate; 11 — death rate; 12 — military events, spends on the war.

The following CM nodes coordinates can be varied as control actions:

- state support of families with children ( $\Delta u_1(k)$ );

- average salary of a worker ( $\Delta u_2(k)$ );
- export volume ( $\Delta u_3(k)$ );
- import volume ( $\Delta u_4(k)$ ).

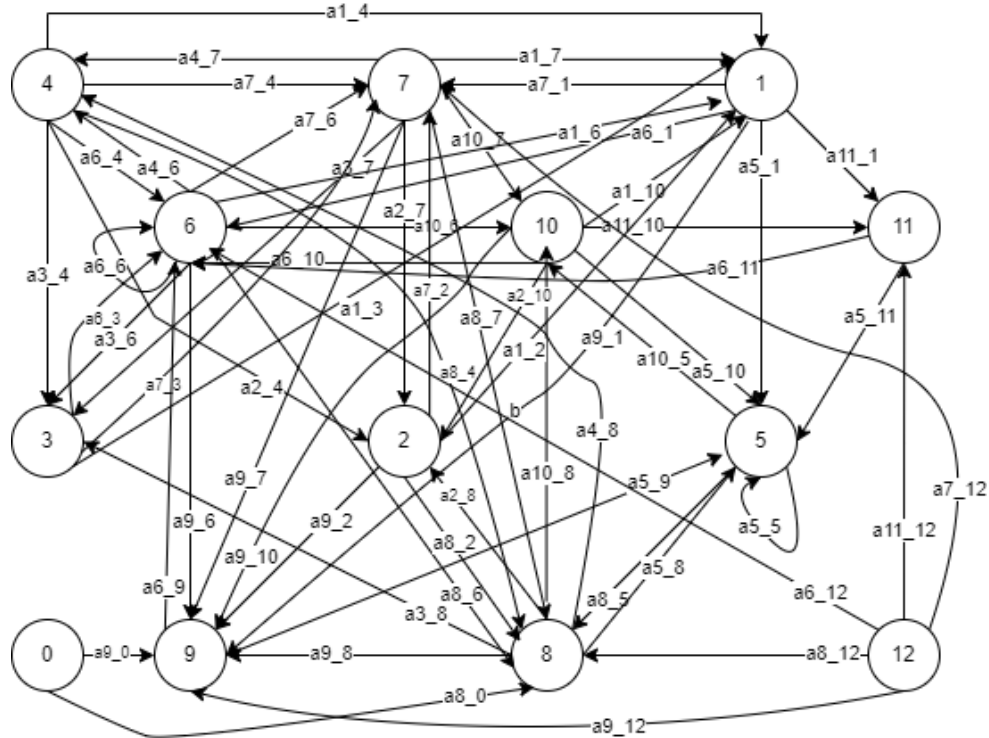


Fig. 1. Demographic security CM

Adjacency matrix  $A$  of the CM impulse process has the following form:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.3 & -0.3 & 0 & 0.6 & 0.65 & 0 & 0 & -0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0.8 & 0.3 & 0 & -0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 & 0.4 & 0 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & -0.7 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 & 0 & 0.1 & 0 & 0 & -0.7 & 0.8 & -0.1 & -0.8 & 0 \\ 0 & 0.7 & 0 & 0.5 & -0.4 & 0 & 0.1 & 0 & 0 & 0.4 & -0.4 & -0.4 & -0.3 \\ 0 & 0.3 & 0.4 & -0.35 & 0.35 & 0 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0.2 \\ -0.3 & 0 & 0.3 & -0.1 & 0.15 & 0.1 & -0.3 & 0.2 & 0 & 0 & 0 & 0 & 0.5 \\ 0.7 & 0.15 & -0.2 & 0 & 0 & 0 & 0.2 & -0.15 & -0.35 & 0 & -0.2 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & -0.45 & 0.5 & -0.15 & 0 & 0 & 0 & 0 \\ 0 & -0.1 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0.3 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Consider main disturbances affecting the demography, to implement control of the demographic security under martial law:

1. Mass migration of population out of Ukraine because of the threat to life under missile attacks on civilian targets in all regions and because of occupation of the territories.

2. High death rate because of military actions at the front and because of missile attacks over all territory of Ukraine.

3. Low birth rate because of young men population in the army, migration, unemployment rate increase and general uncertainty about the future affecting willingness of people to have children.

All these disturbances are practically impossible to describe mathematically using probabilistic indicators, specifically, to find their distributions, research their stationarity, analyse their fluctuations calculating their variance, find correlations etc. We can only set up limitations on their amplitude when describing the disturbances.

### **PROBLEM OF SUPPRESSING CONSTRAINED INTERNAL AND EXTERNAL DISTURBANCES DURING CONTROL OF DEMOGRAPHIC SECURITY COGNITIVE MAP IMPULSE PROCESS**

Studies [3–5] present the theoretical foundations on the suppression of arbitrary constrained external disturbances in terms of invariant ellipsoids based on the design of a static state feedback, which minimizes the size of the invariant ellipsoid of the dynamical system. In this case, we implemented a robust control, where the analysis and synthesis problems are reduced to equivalent conditions in the form of linear matrix inequalities (LMI), solved numerically on the basis of semi-definite programming. In [5] we solve the problem of suppression of constrained external disturbances based on the invariant ellipsoids approach in the implementation of a closed-loop control system of impulse processes in CM of cryptocurrency on financial markets.

The general model of the dynamics of impulse processes (2) is decomposed into two interrelated systems of difference equations:

$$\Delta \bar{X}(k+1) = A_1 \Delta \bar{X}(k) + D \Delta \bar{Z}(k); \quad (5)$$

$$\Delta \bar{Z}(k+1) = A_2 \Delta \bar{Z}(k) + \Psi \Delta \bar{X}(k). \quad (6)$$

Here  $\bar{X}$  is the vector of measurable coordinates of CM nodes which are to be stabilized later;  $\bar{Z}$  is the vector of CM coordinates considered as disturbances. The matrices  $A_1$ ,  $D$ ,  $A_2$ ,  $\Psi$  are parts of the adjacency matrix of the initial model (2). The matrices  $D$ ,  $\Psi$  show the relationships between the first (5) and the second (6) parts of the initial CM (2). The increments of coordinates  $\Delta \bar{Z}(k)$  are taken into account as external constrained disturbances with unknown probabilistic characteristics in the first system of equations (5) of the CM model.

We designed a control vector to suppress constrained perturbations  $\Delta \bar{Z}(k)$  by implementing static state controller in the feedback loop

$$\Delta \bar{U}(k) = -K_p \Delta \bar{X}(k), \quad (7)$$

which acts directly on the measured nodes coordinates  $\bar{X}$  of the first impulse process equations system (5) according to the controlled model:

$$\Delta \bar{X}(k+1) = A_1 \Delta \bar{X}(k) + B \Delta \bar{U}(k) + D \Delta \bar{Z}(k) \quad (8)$$

The control is performed by changing the resources of the CM nodes, which are affected by the vector  $\Delta \bar{U}(k)$ .

In this paper, the change in the weight coefficients  $\Delta A_1(k)$  with respect to the known estimated values of the matrix  $\hat{A}_1$  is proposed to be considered as the internal perturbations in the CM impulse process model (5) of the demographic situation. For this purpose, in [4, 5] we modify the model (5) as follows:

$$\Delta \bar{X}(k+1) = A_1 \Delta \bar{X}(k) + \Delta A_1 \Delta \bar{X}(k) + D \Delta \bar{Z}(k), \quad (9)$$

where  $\Delta A_1 = A_1 - A_{1_{\text{var}}}(k)$  is the change in the adjacency matrix of CM (5) during the sampling period,  $A_{1_{\text{var}}}(k)$  is the real unknown value of the matrix  $A_1$ , which changes as the demographic system evolves.

Let us denote the increment of internal perturbations in (9) as  $\Delta A_1(k) \Delta y(k) = \Delta \bar{w}(k)$ . Then the equation of the uncontrolled impulse process (9) will be written as:

$$\Delta \bar{X}(k+1) = A_1 \Delta \bar{X}(k) + (I_1 \ D) \begin{bmatrix} \Delta \bar{w}(k) \\ \Delta \bar{Z}(k) \end{bmatrix}, \quad (10)$$

where the vectors and matrices have the following dimensions:  $\dim \Delta \bar{X} = n$ ;  $\dim \Delta \bar{Z} = p$ ;  $\dim \Delta \bar{w} = n$ ;  $A_1 (n \times n)$ ;  $D (n \times p)$ ,  $I$  is a unit matrix of dimension  $n \times n$ . We assume that the internal and external perturbations are jointly constrained by the norm  $l_\infty$ , so that:

$$\left\| \begin{bmatrix} \Delta \bar{w}(k) \\ \Delta \bar{Z}(k) \end{bmatrix} \right\|_\infty = \sup \left\{ \left[ \Delta \bar{w}^T(k) \ \Delta \bar{Z}^T(k) \right] \begin{bmatrix} \Delta \bar{w}(k) \\ \Delta \bar{Z}(k) \end{bmatrix} \right\}^{1/2} \leq 1. \quad (11)$$

In [3] invariant ellipsoids on state variables are proposed to describe the characteristic of the effect of disturbances of the type (11) on the trajectory of a dynamic discrete system (10). For the CM they take the form:

$$\varepsilon_{\Delta \bar{X}} = \{ \Delta \bar{X}(k) \in R^n : \Delta \bar{X}^T P^{-1} \Delta \bar{X} \leq 1 \}, \quad P > 0, \quad (12)$$

if from  $\Delta \bar{X}(0) \in \varepsilon_{\Delta \bar{X}}$  the condition  $\Delta \bar{X}(k) \in \varepsilon_{\Delta \bar{X}}$  follows for all discrete moments of time  $k = 1, 2, 3, \dots$ . Then the matrix  $P$  is called the matrix of the ellipsoid  $\varepsilon_{\Delta \bar{X}}$ .

In [4; 5] the condition of invariance of the ellipsoid (12) under disturbances (11) is proven. According to it, invariance is guaranteed when the following LMI is met:

$$\frac{1}{\alpha} A_1 P A_1^T - P + \frac{I_1 + D D^T}{(1-\alpha)} \leq 0. \quad (13)$$

### ALGORITHM FOR A STATE CONTROLLER DESIGN FOR THE COGNITIVE MAP IMPULSE PROCESS

The state equation of the controlled CM impulse process (10) under additional internal disturbances  $\Delta w(k)$  takes the form:

$$\Delta \bar{X}(k+1) = A_1 \Delta \bar{X}(k) + B \Delta \bar{U}(k) + (I_1 \ D) \begin{bmatrix} \Delta \bar{w}(k) \\ \Delta \bar{Z}(k) \end{bmatrix}. \quad (14)$$

When the state controller (7) is applied, the equation of the closed-loop CM impulse process control system is written as follows:

$$\Delta\bar{X}(k+1) = (A_1 - BK_p)\Delta\bar{X}(k) + (I_1 - D) \begin{bmatrix} \Delta\bar{w}(k) \\ \Delta\bar{Z}(k) \end{bmatrix}. \quad (15)$$

It is assumed that the pair  $(A_1 - B)$  in the model (14) is controllable. Then the LMI (13) for the closed-loop system looks like:

$$\frac{1}{\alpha}(A_1 - BK_p)P(A_1 - BK_p)^T - P + \frac{I_1 + DD^T}{(1-\alpha)} \leq 0. \quad (16)$$

We consider the minimization of the trace of the ellipsoid matrix (12) as the optimality criterion for the design of the controller (7):

$$\text{tr}P(\alpha) \rightarrow \min, \quad \alpha^* \leq \alpha < 1, \quad (17)$$

This ensures minimization of the size of the invariant ellipsoid (12) with the largest suppression of disturbances  $\begin{bmatrix} \Delta\bar{w}(k) \\ \Delta\bar{Z}(k) \end{bmatrix}$ , which are constrained only by the maximum range (11). After multiplying the factors in the inequality (16), we obtain:

$$\frac{1}{\alpha}(A_1PA_1^T - BK_pPA_1^T - A_1PK_p^TB^T + BK_pPK_p^TB^T) - P + \frac{I_1 + DD^T}{(1-\alpha)} \leq 0. \quad (18)$$

Inequality (18) is nonlinear with respect to  $P$  and  $K_p$ , which need to be optimized. In [3] a replacement  $L = K_pP$  and introduction of an additional constraint is done:

$$\begin{bmatrix} R & L \\ L^T & P \end{bmatrix} \geq 0, \quad (19)$$

where  $R = R^T$ . This inequality is equivalent to  $R \geq LP^{-1}L^T = K_pPK_p^T$  according to the Schur's formula at  $P > 0$ . Then to meet inequality (18) it is sufficient that:

$$\frac{1}{\alpha}(A_1PA_1^T - BLA_1^T - A_1L^TB^T + BRB^T) - P + \frac{I_1 + DD^T}{(1-\alpha)} \leq 0. \quad (20)$$

Minimization of criterion (17) under constraints (19), (20) is performed with respect to variables  $P, L, R$  using semi-definite programming method by using Matlab-based SeDuMi Toolbox. Then the matrix  $\hat{K}_p$  of the optimal state controller (7) is defined as:

$$\hat{K}_p = \hat{L}\hat{P}^{-1} \quad (21)$$

with the estimated values of  $\hat{\alpha}, \hat{P}, \hat{L}, \hat{R}$ , providing minimization of criterion (17) under constraints (19), (20).

### **PROBLEM OF THE COGNITIVE MAP WEIGHTS IDENTIFICATION BASED ON THE RECURRENT LEAST SQUARES METHOD**

The model of the controlled CM impulse process (3) of the "input-output" type can be represented as:

$$(I - A_1q^{-1})\Delta\bar{Y}(k) = Bq^{-1}\Delta\bar{U}(k). \quad (22)$$

Weighting coefficients of the adjacency matrix  $A$  are usually determined by applying expert estimates based on cause-and-effect relations. In the process of evolving of the demographic situation, these coefficients in the model (22) will change over time, depending on changes in the influence of the CM nodes on each other. So the problem of adaptive control of the CM impulse process appears, when both estimation of the parameters (coefficients) of the adjacency matrix  $A_1$  and design of the control vector  $\bar{U}(k)$  must be performed simultaneously.

Let us describe the equation (22) coordinate-wise (for each CM node):

$$\Delta y_i(k) = \sum_{j=1}^n a_{ij} \Delta y_j(k-1) + b_i \Delta u_i(k-1) + \xi_i(k). \quad (23)$$

It is assumed that the disturbances  $\xi_i(k)$ , caused by inaccurate measuring of the CM nodes coordinates and inaccurate knowledge of the model coefficients, are white noise. This assumption is plausible because  $y_i(k), u_i(k)$  in model (23) are presented in the form of the first differences, i.e. increments. It should also be taken into account that the structure of the matrix  $A_1$  is known and some of the coefficients  $a_{ij}$  are obviously equal to zero (in those cases when there are no connections between the corresponding CM nodes).

Let us write model (23) as follows:

$$\Delta y_i(k) - b_i \Delta u_i(k-1) = \bar{X}_i^T(k) \bar{\Theta}_i + \xi_i(k), \quad (24)$$

where  $\bar{\Theta}_i = [a_{ij_1} \dots a_{ij_{P_i}}]^T$  consists of the non-zero coefficients in the  $i$ -th row of matrix  $A_1$ ,  $\bar{X}_i^T(k) = [\Delta Y_{j_1}(k-1), \dots, \Delta Y_{j_{P_i}}(k-1)]$  is a vector of measured CM nodes coordinates.

The current estimate of the vector  $\bar{\Theta}_i$  is denoted by  $\hat{\Theta}_i(k)$ . To estimate the weight coefficients of the matrix  $A_1$  we apply the recurrent least squares method [6–9]:

$$\begin{aligned} \hat{\Theta}_i(k) &= \hat{\Theta}_i(k-1) + K_i(k) (\Delta y_i(k) - b_i \Delta u_i(k-1) - \bar{X}_i^T(k) \hat{\Theta}_i(k-1)); \\ K_i(k) &= \frac{1}{1 + \bar{X}_i^T(k) P_i(k-1) \bar{X}_i(k)} P_i(k-1) \bar{X}_i(k) \bar{X}_i^T(k) P_i(k-1); \\ P_i(k) &= P_i(k-1) - \frac{1}{1 + \bar{X}_i^T(k) P_i(k-1) \bar{X}_i(k)} P_i(k-1) \bar{X}_i(k) \bar{X}_i^T(k) P_i(k-1). \end{aligned} \quad (25)$$

The recurrent procedure (25) should be performed for each CM node  $\Delta y_i(k)$ ,  $i=1,2,\dots,n$  at each sampling period. We use the obtained estimates  $\hat{\Theta}_i(k)$  as the coefficients values of the adjacency matrix  $A_1$  at the current sampling period in the control algorithm (7), (20), (21). For parametric identification of the adjacency matrix  $A_1$ , we can also apply non-recurrent identification methods outlined in [10].

#### EXPERIMENTAL RESEARCH OF THE SYSTEM SUPPRESSING CONSTRAINED INTERNAL AND EXTERNAL DISTURBANCES DURING CONTROL OF THE DEMOGRAPHIC SECURITY COGNITIVE MAP IMPULSE PROCESS

To ensure demographic security in Ukraine it is reasonable to stabilize the following coordinates  $\bar{X}$  of the CM on Fig. 1: 0, 1, 2, 3, 4, 5, 6, 9. The following coor-

dinates  $\bar{Z}$  are considered as disturbances affecting the demographic security: 7, 8, 10, 11, 12. After decomposition of the model (2) into models (5) and (6) we conclude that although model (2) is unstable, state equations (5) and (6) are stable. Control actions  $\Delta u_1(k)$ ,  $\Delta u_2(k)$ ,  $\Delta u_3(k)$ ,  $\Delta u_4(k)$  are fed to the nodes 0, 1, 3, 4 respectively. So matrix  $B$  in the controlled impulse process equation (8) is the following:

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}^T$$

During simulation of closed-loop control system dynamics of the CM impulse process based on the proposed method, we applied step impulse with unit amplitude as an external disturbance at the initial time moment fed to the node (12) — military events, spends on the war. Internal disturbances are generated as following: at each sampling period non-zero coefficients of the matrix  $A_1$  are varied under the formula  $A_{1\text{var}}(k) = A_1 \xi(k)$ , where  $\xi(k)$  is a normal random variable (Gaussian white noise) for the control only values of  $A_1$  are used while  $A_{1\text{var}}$  are applied as unknown internal disturbance. Initial levels of all the CM nodes coordinates are taken equal to zero for simplicity.

Fig. 2 shows the transient processes of the CM nodes coordinates, Fig. 3 — their increments. Here solid lines denote transient process under control, dashed lines — without control. Fig. 4 shows control actions changes.

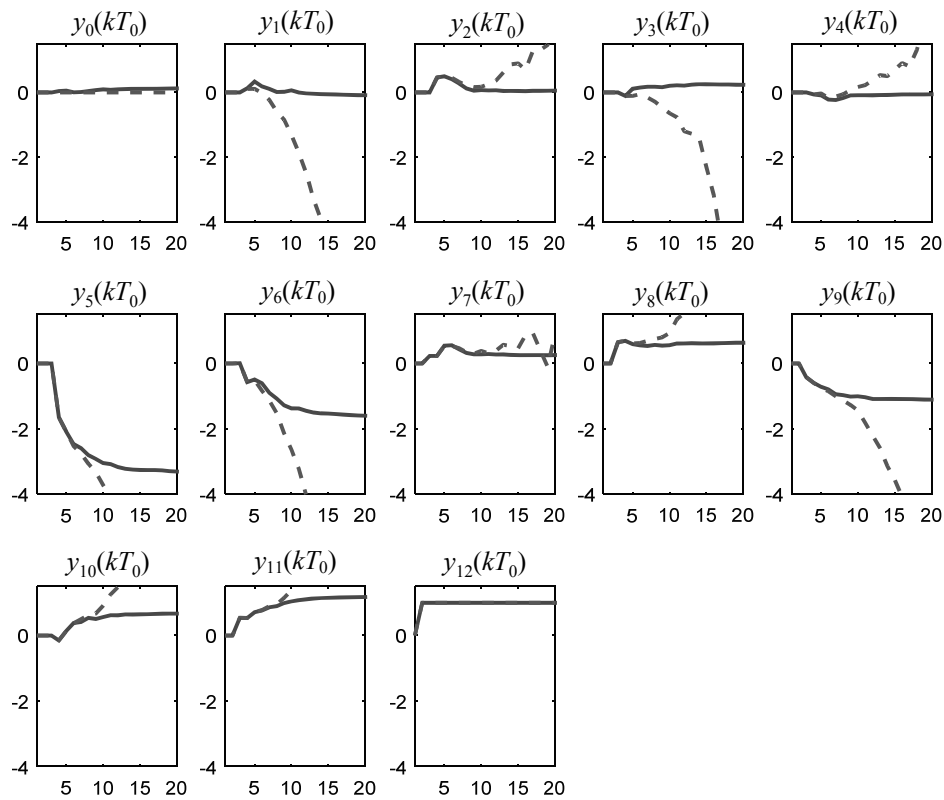


Fig. 2. CM nodes coordinates

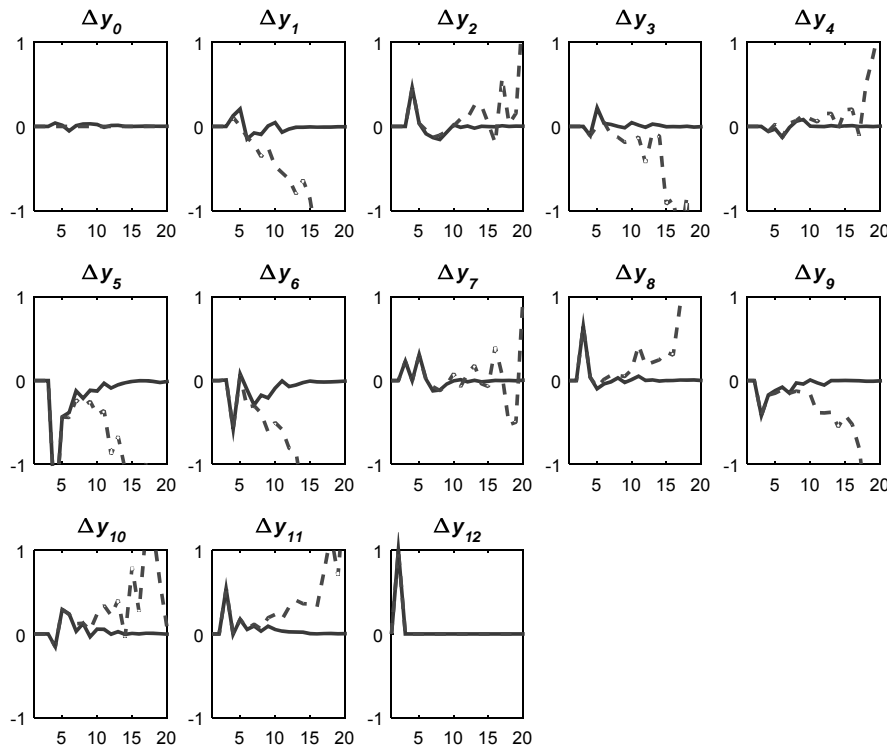


Fig. 3. CM nodes coordinates increments

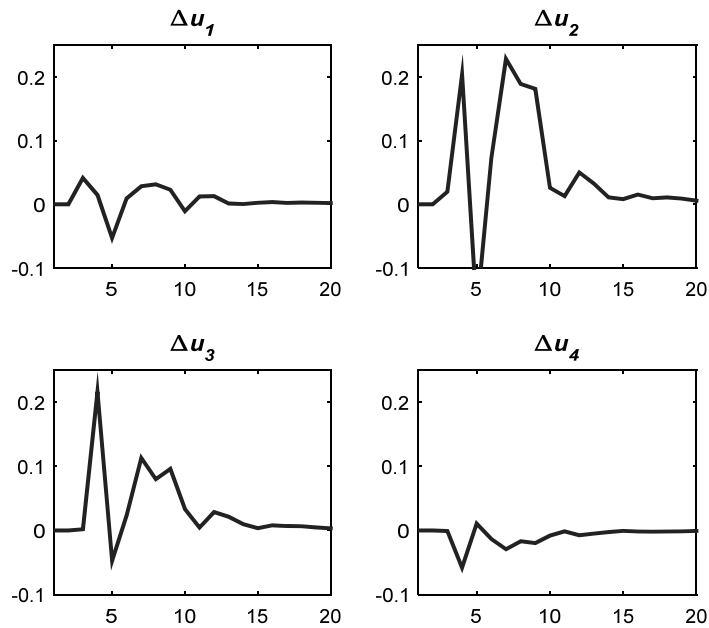


Fig. 4. Control actions

**CONCLUSIONS**

The paper considers important problem of demographic security under martial law in Ukraine. Possible approach to solve this problem was suggested based on

the CM impulse processes modelling and control. Specifically, the CM of demographic security was created and the control method for suppressing disturbances based on invariant ellipsoids was applied. As a result, the control system was designed and the simulation was performed.

Based on the simulation results, we can conclude that the suggested approach will help to stabilize very dangerous and unstable demographic process initiated by increase of the military spends and the military events intensity. Without control this process leads to the catastrophic depopulation of Ukraine. Under the suggested control, the simulation demonstrates that despite significant decrease of the population at the beginning, we are able to stabilize it at some level and stop this process. Main control actions the government should apply are: export increase and import decrease, average salary increase and support of the families with children. The latter two of them are necessary to increase birth rate and decrease migration, while the former two actions are necessary to prevent inflation and stabilize economy.

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**ЗАДАЧІ АВТОМАТИЗОВАНОГО КЕРУВАННЯ ДИНАМІЧНИМИ ПРОЦЕСАМИ СИСТЕМНОГО ЗАБЕЗПЕЧЕННЯ БЕЗПЕКИ УКРАЇНИ НА ОСНОВІ МОДЕЛЕЙ ІМПУЛЬСНИХ ПРОЦЕСІВ У КОГНІТИВНИХ КАРТАХ. ЧАСТИНА 1. ЗАБЕЗПЕЧЕННЯ ДЕМОГРАФІЧНОЇ БЕЗПЕКИ / В.Д. Романенко, Ю.Л. Мілявський**

**Анотація.** Наведено когнітивну карту (КК) демографічної безпеки, на основі якої описано динамічну модель імпульсних процесів КК у вигляді системи різницевих рівнянь (рівнянь Робертса). Виконано вибір зовнішнього вектора керувальних дій імпульсним процесом КК, який реалізується шляхом варіювання координат вершин КК. Реалізовано замкнену систему керування імпульсним процесом КК, до складу якої входить синтезований на основі методів теорії автоматичного керування багатовимірний дискретний регулятор, який формує вибрані керувальні дії. Розв’язано задачу проектування дискретного регулятора для автоматизованого керування динамічними процесами для забезпечення демографічної безпеки. Функція регулятора полягає у приглушенні зовнішніх та внутрішніх збурень при керуванні імпульсними процесами КК на основі методу інваріантних еліпсоїдів. Наведено алгоритм ідентифікації вагових коефіцієнтів КК на основі рекурентного методу найменших квадратів. Подано результати дослідження якості динамічних процесів стосовно забезпечення демографічної безпеки в Україні в разі дії різноманітних збурень в умовах воєнного стану.

**Ключові слова:** когнітивна карта, демографічна безпека, інваріантний еліпсоїд, лінійні матричні нерівності, імпульсний процес.