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A PROBABILISTIC MODEL OF THE COLONEL BLOTTO GAME WITHOUT SYMMETRY AND HOMOGENEITY CONSTRAINTS

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Abstract. The classic Colonel Blotto game for two players was considered. The probabilistic model of the payoff functions of the specified problem was investigated, and the game conditions are not subject to the restrictions of symmetry and homogeneity. The system of equations obtained using the method of Lagrange multipliers has a large dimension. In order to find a solution, a way to reduce the dimension was found. The found ratio between the resources of both players, distributed over the courts, made it possible to identify a parameter determined by the ratio of Lagrange multipliers from the corresponding functions for both players. For such a parameter, an interval constraint that it satisfies was found, and an equation is formulated to find it, which is solved numerically. The found value of the parameter makes it possible to calculate individual Lagrange multipliers and obtain the optimal distribution of players' resources in the form of a Nash equilibrium in pure game strategies. An example of a game under significantly different conditions for players was studied.

Keywords: conflict confrontation, optimal allocation of resources, two-person game, Colonel Blotto's game, probabilistic payoff model, Nash equilibrium, efficiency of resource use, 1-parametrization.

INTRODUCTION

Colonel Blotto's problem as a model of competitive struggle between two players on several platforms has long become a classic. It was first presented by Borel [1] in 1921, but despite its century-old history of applications, it still remains relevant and attracts the attention of researchers [2]. As a model of competitive struggle, it finds numerous applications in the social, economic spheres of activity, etc.

The game is described as follows. Two players compete against each other on several sites, which can be battlefields, election areas, product markets, etc. Each participant has a limited resource that must be distributed across the sites in such a way as to maximize their winnings, taking into account that each site has its own value for each of the rivals. In the auction model, the winner of a given site is the player who has allocated more resources to it than the opponent. In the case of equal allocated resources, a draw occurs.

More interesting for us is the case of the probabilistic model. Here, the probability of winning on a certain site is directly proportional to the resource allocated to it and inversely proportional to the sum of the resources allocated to it by both players. It should be noted that the following constraints are usually imposed on the game: symmetry, when the total resources of each player are the same, and homogeneity, when the worth of victory for any site coincides with that of its opponent. Thus, when searching for equilibrium in Colonel Blotto's problem, researchers limited themselves to symmetric or homogeneous cases, with the latter condition being used quite often [3–5].

Hence, the case where these two conditions are not met is of not only scientific interest. When trying to find equilibrium strategies for such a problem, it becomes necessary to solve systems of equations of high dimension. Using the approach proposed in [6], the system of equations is reduced to the one-dimensional case and represents by a single variable equation. In this case, we arrive at the precize solution of the asymmetric Colonel Blotto game without homogeneity constraints.

PROBLEM STATEMENT

Consider the probabilistic model of Colonel Blotto's game. Two opposing parties distribute their resources x_i and y_i, $i = \overline{1,n}$, across n sites. The resource constraints are determined by the following inequalities, $\sum_{i=1}^{n} x_i \le R_x$, $\sum_{i=1}^{n} y_i \le R_y$. The probability of the first player winning on the i-th court is given as follows:

$$p_i^x(x_i, y_i) = \frac{\alpha_i x_i^{r_i}}{\alpha_i x_i^{r_i} + \beta_i y_i^{r_i}},$$

where $r_i \in (0,1]$, and $\alpha_i > 0$, $\beta_i > 0$ are the efficiency coefficients of resource use on the corresponding sites for the first and second players. Similarly, the formula for the probability of winning $p_i^y(x_i, y_i)$ for the second player is:

$$p_i^y(x_i, y_i) = \frac{\beta_i y_i^{r_i}}{\alpha_i x_i^{r_i} + \beta_i y_i^{r_i}}.$$

In this case, the payoff functions take the form:

$$F_x(x, y) = \sum_{i=1}^{n} X_i p_i^x(x_i, y_i),$$

$$F_y(x, y) = \sum_{i=1}^{n} Y_i p_i^y(x_i, y_i),$$

where X_i, Y_i is the value of winning on the i-th court for each of the two players. A Nash equilibrium in pure strategies (x^*, y^*) is a pair of vectors that satisfies for any (x, y) satisfies: $F_x(x^*, y^*) \ge F_x(x^*, y)$, $F_y(x^*, y^*) \ge F_y(x, y^*)$.

Asymmetric game when $R_x \neq R_y$, with the following parameter values $r_i = 1$, $\alpha_i = 1$, $i \in N$ was considered in [3] for the case $X_i = Y_i = const$. The only Nash equilibrium in this case is the use of such pure strategies, when resources must be evenly distributed between sites. A continuation of these studies was the work [4], where an equilibrium in pure strategies was found for the case $X_i = Y_i = V_i$ with arbitrary $r_i \in (0,1]$, $\alpha_i > 0$, $i \in N$. This paper considers the situation when $X_i \neq Y_i$.

The aim of the work is to find the Nash equilibrium in pure strategies for the asymmetric and heterogeneous Colonel Blotto game for the case of the probabilistic model.

PROCEDURE FOR CONSTRUCTING AN OPTIMAL SOLUTION

To find a solution to the problem, we write the Lagrange function for each of the players:

$$L_x = F_x + \lambda_x (R_z - \sum_{i=1}^{n} x_i),$$

$$L_y = F_y + \lambda_y (R_y - \sum_{i=1}^{n} y_i).$$

Then we get the system of equations:

$$\frac{\partial L_x}{\partial x_i} = \frac{\partial F_x}{\partial x_i} - \lambda_x = 0,$$

$$\frac{\partial L_y}{\partial y_i} = \frac{\partial F_y}{\partial y_i} - \lambda_y = 0,$$

$$\frac{\partial L_x}{\partial \lambda_x} = R_x - \sum_{i-1}^{n} x_i = 0,$$

$$\frac{\partial L_y}{\partial \lambda_y} = R_y - \sum_{i-1}^{n} y_i = 0.$$

Starting from the system, we get:

$$\frac{\partial L_x}{\partial x_i} = X_i \frac{\alpha_i \beta_i r_i x_i^{r_i - 1} y_i^{r_i}}{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2} = \lambda_x,$$

$$\frac{\partial L_y}{\partial y_i} = Y_i \frac{\alpha_i \beta_i r_i x_i^{r_i} y_i^{r_i - 1}}{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2} = \lambda_y.$$

We have $(2n+2)$ equations with $(2n+2)$ variables: $x_1, \ldots, x_n, y_1, \ldots, y_n, \lambda_x, \lambda_y$, that is, the system can be solved. Consider the ratio $\frac{\lambda_x}{\lambda_y}$. We will get:

$$\frac{\lambda_x}{\lambda_y} = X_i \frac{\alpha_i \beta_i r_i x_i^{r_i - 1} y_i^{r_i}}{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2} \cdot \frac{1}{Y_i} \frac{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2}{\alpha_i \beta_i r_i x_i^{r_i} y_i^{r_i - 1}} = \frac{X_i}{Y_i} \cdot \frac{y_i}{x_i}.$$

From here $y_i = \frac{\lambda_x}{\lambda_y}\cdot\frac{Y_i}{X_i}x_i$. Let's mark $\lambda = \frac{\lambda_x}{\lambda_y}$, $C_i = \frac{Y_i}{X_i}$. Then

$$\frac{y_i}{x_i} = \lambda C_i . \tag{1}$$

We will use expression (1) to go from the multidimensional case with $(2n+2)$ equations and $(2n+2)$ variables to the one-dimensional case. To do this, we will find an equation for searching λ. Consider the expressions $R_x\lambda_x$ and $R_y\lambda_y$:

$$\begin{aligned} R_x\lambda_x &= \sum_{i=1}^{n} x_i\lambda_x = \sum_{i=1}^{n} x_i X_i \frac{\alpha_i\beta_i r_i x_i^{r_i-1} y_i^{r_i}}{(\alpha_i x_i^{r_i}+\beta_i y_i^{r_i})^2} = \\ &= \sum_{i=1}^{n} X_i \frac{\alpha_i\beta_i r_i x_i^{r_i} y_i^{r_i}}{(\alpha_i x_i^{r_i}+\beta_i y_i^{r_i})^2}\cdot\frac{/(x_i^{r_i})^2}{/(x_i^{r_i})^2} = \sum_{i=1}^{n} X_i \frac{\alpha_i\beta_i r_i (\frac{y_i}{x_i})^{r_i}}{(\alpha_i+\beta_i(\frac{y_i}{x_i})^{r_i})^2} = \\ &= \sum_{i=1}^{n} X_i \frac{\alpha_i\beta_i r_i(\lambda C_i)^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2}. \end{aligned} \tag{2}$$

Similarly,

$$R_y\lambda_y = \sum_{i=1}^{n} Y_i \frac{\alpha_i\beta_i r_i(\lambda C_i)^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2} . \tag{3}$$

From formulas (2) and (3) we determine the relationship between $R_x\lambda_x$ and $R_y\lambda_y$:

$$\begin{aligned} \frac{R_x\lambda_x}{R_y\lambda_y} = \frac{R_x}{R_y}\lambda &= \sum_{i=1}^{n} X_i \frac{\alpha_i\beta_i r_i(\lambda C_i)^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2} \Big/ \sum_{i=1}^{n} Y_i \frac{\alpha_i\beta_i r_i(\lambda C_i)^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2} = \\ &= \sum_{i=1}^{n} X_i \frac{\alpha_i\beta_i r_i C_i^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2} \Big/ \sum_{i=1}^{n} Y_i \frac{\alpha_i\beta_i r_i C_i^{r_i}}{(\alpha_i+\beta_i(\lambda C_i)^{r_i})^2} . \end{aligned}$$

Let's rewrite this formula in the following form:

$$\lambda\frac{R_x}{R_y}\sum_{i=1}^{n} Y_i \frac{\frac{\alpha_i}{\beta_i} r_i C_i^{r_i}}{(\frac{\alpha_i}{\beta_i}+(\lambda C_i)^{r_i})^2} = \sum_{i=1}^{n} X_i \frac{\frac{\alpha_i}{\beta_i} r_i C_i^{r_i}}{(\frac{\alpha_i}{\beta_i}+(\lambda C_i)^{r_i})^2} . \tag{4}$$

Let's find the limits within which the value lies λ. From (1) we have $y_i = \lambda C_i x_i$. Then $\sum_{i=1}^{n} y_i = R_y = \lambda\sum_{i=1}^{n} C_i x_i$. Hence

$$\min_i C_i \cdot \sum_{i=1}^{n} x_i \le \sum_{i=1}^{n} C_i x_i \le \max_i C_i \cdot \sum_{i=1}^{n} x_i .$$

Since $R_x = \sum_{i=1}^{n} x_i$, and $\sum_{i=1}^{n} C_i x_i = \frac{R_y}{\lambda}$, then

$$R_x \min_i C_i \le \frac{R_y}{\lambda} \le R_x \max_i C_i ,$$

$$\frac{R_y}{R_x} (\max_i C_i)^{-1} \le \lambda \le \frac{R_y}{R_x} (\min_i C_i)^{-1} ,$$

$$\frac{R_y}{R_x} \min_i \frac{X_i}{Y_i} \le \lambda \le \frac{R_y}{R_x} \max_i \frac{X_i}{Y_i} .$$

Since, except for λ, all other parameters of equation (4) are known, its solution can be found on the specified segment by an appropriate numerical method, for example, the dichotomy method.

Let's find the optimal values for x_i^* , y_i^* .

$$\lambda_x x_i = X_i \frac{\alpha_i \beta_i r_i x_i^{r_i - 1} y_i^{r_i}}{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2} x_i = X_i \frac{\alpha_i \beta_i r_i x_i^{r_i} y_i^{r_i}}{(\alpha_i x_i^{r_i} + \beta_i y_i^{r_i})^2} \cdot \frac{/(\beta_i x_i^{r_i})^2}{/(\beta_i x_i^{r_i})^2} =$$

$$= X_i \frac{\frac{\alpha_i}{\beta_i} r_i (\frac{y_i}{x_i})^{r_i}}{(\frac{\alpha_i}{\beta_i} + (\frac{y_i}{x_i})^{r_i})^2} = X_i \frac{\frac{\alpha_i}{\beta_i} r_i (\lambda C_i)^{r_i}}{(\frac{\alpha_i}{\beta_i} + (\lambda C_i)^{r_i})^2} = a_i .$$

Similarly,

$$\lambda_y y_i = Y_i \frac{\frac{\alpha_i}{\beta_i} r_i (\lambda C_i)^{r_i}}{(\frac{\alpha_i}{\beta_i} + (\lambda C_i)^{r_i})^2} = b_i .$$

In the right-hand sides of the equalities, all parameters are known, therefore a_i and b_i take specific values. Now we express λ_x in terms of R_x and a_i, and λ_y in terms of R_y and b_i, after which we find x_i^* and y_i^*.

$$\lambda_x x_i = a_i \;\Rightarrow\; \sum_{i=1}^{n} \lambda_x x_i = \sum_{i=1}^{n} a_i \;\Rightarrow\; \lambda_x \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} a_i \;\Rightarrow$$

$$\lambda_x R_x = \sum_{i=1}^{n} a_i \;\Rightarrow\; \lambda_x^{-1} = \frac{R_x}{\sum_{i=1}^{n} a_i} .$$

$$x_i^* = \frac{a_i}{\lambda_x} = \frac{R_x a_i}{\sum_{i=1}^{n} a_i} . \qquad (5)$$

Similarly, for λ_y:

$$\lambda_y y_i = b_i \Rightarrow \lambda_y^{-1} = \frac{R_y}{\sum_{i=1}^{n} b_i}.$$

$$y_i^* = \frac{b_i}{\lambda_y} = \frac{R_y b_i}{\sum_{i=1}^{n} b_i}. \tag{6}$$

Formulas (5) and (6) express the optimal solution to the problem — the Nash equilibrium in pure strategies. Then the payoff functions take the following form:

$$F_x(x,y) = \sum_{i=1}^{n} X_i \frac{\alpha_i x_i^{r_i}}{\alpha_i x_i^{r_i} + \beta_i y_i^{r_i}} = \sum_{i=1}^{n} X_i \frac{\frac{\alpha_i}{\beta_i}}{\frac{\alpha_i}{\beta_i} + (\lambda C_i)^{r_i}},$$

$$F_y(x,y) = \sum_{i=1}^{n} Y_i \frac{\beta_i y_i^{r_i}}{\alpha_i x_i^{r_i} + \beta_i y_i^{r_i}} = \sum_{i=1}^{n} Y_i \frac{(\lambda C_i)^{r_i}}{\frac{\alpha_i}{\beta_i} + (\lambda C_i)^{r_i}}.$$

RESULTS OF THE NUMERICAL EXPERIMENT

We will conduct a numerical experiment based on theoretical calculations. We will consider Colonel Blotto's game on five platforms and also record the values of some parameters:

$R_x = 100$.

$r_1 = 0.1$; $r_2 = 0.25$; $r_3 = 0.5$; $r_4 = 0.75$; $r_5 = 1$.

$\alpha_1 = 1.4$; $\alpha_2 = 1.9$; $\alpha_3 = 3.1$; $\alpha_4 = 3.7$; $\alpha_5 = 4.1$.

$\beta_1 = 1.3$; $\beta_2 = 1.8$; $\beta_3 = 2.8$; $\beta_4 = 3.5$; $\beta_5 = 5$.

We will consider three cases, where $R_y = 150$, $R_y = 250$, $R_y = 350$. For each case, we will calculate two options.

The first case.

Table 1. Option No.1 ($R_y = 150$)

i	1	2	3	4	5
X_i / Y_i	1.1/1.5	1.1/2.1	1.1/3.4	1.1/4.2	1.1/4.9
x_i^* / y_i^*	5.01/2.96	12.56/10.38	24.22/32.42	29.02/47.98	29.16/56.24
p_i^x / p_i^y	0.53/0.46	0.52/0.47	0.48/0.51	0.42/0.57	0.29/0.7
F_x / F_y	2.44/9.22				

Calculated value $\lambda = 0.407$.

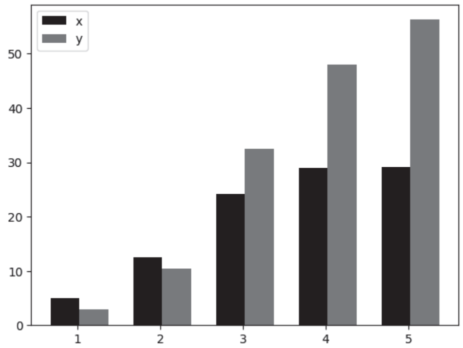

Fig. 1. Optimal solutions of players

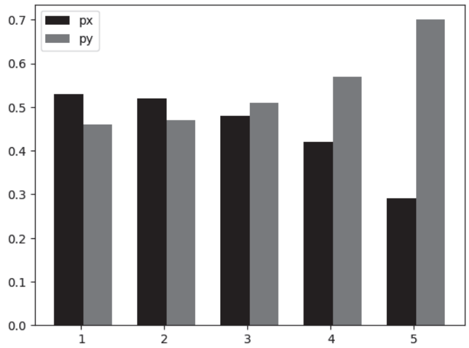

Fig. 2. Probabilities of players' winnings

Table 2. Option No. 2 ($R_y = 150$)

i	1	2	3	4	5
X_i / Y_i	5.5/1.5	5.5/2.1	5.5/3.4	5.5/4.2	5.5/4.9
x_i^* / y_i^*	3.92/2.19	9.18/7.19	18.69/23.68	29.11/45.57	39.07/71.35
p_i^x / p_i^y	0.53/0.46	0.52/0.47	0.49/0.5	0.43/0.56	0.3/0.69
F_x / F_y	12.47/9.1				

Calculated value $\lambda = 2.171$.

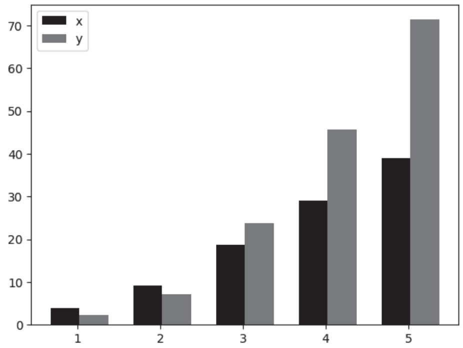

Fig. 3. Optimal solutions of players

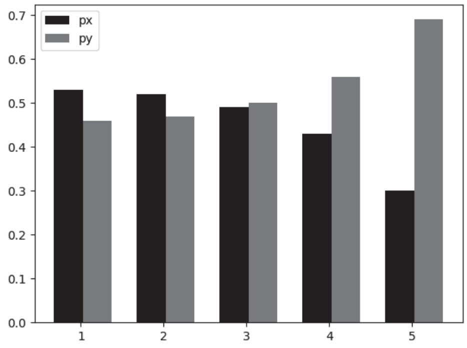

Fig. 4. Probabilities of players' winnings

The second case.

Table 3. Option No. 1 ($R_y = 250$)

i	1	2	3	4	5
X_i / Y_i	1.1/1.5	1.1/2.1	1.1/3.4	1.1/4.2	1.1/4.9
x_i^* / y_i^*	4.35/4.16	10.76/14.41	21.69/47.04	29.36/78.66	33.82/105.7
p_i^x / p_i^y	0.51/0.48	0.49/0.5	0.42/0.57	0.33/0.66	0.2/0.79
F_x / F_y	2.13/10.34				

Calculated value $\lambda = 0.684$.

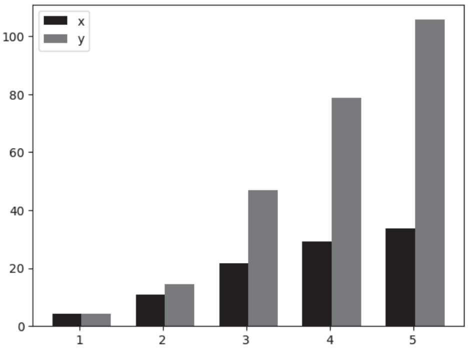

Fig. 5. Optimal solutions of players

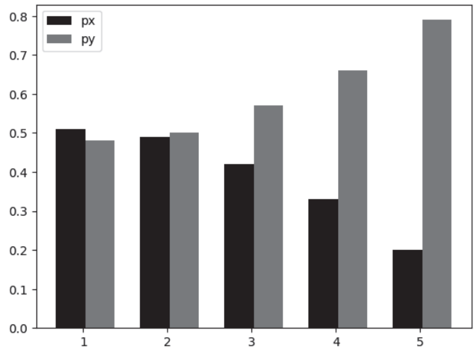

Fig. 6. Probabilities of players' winnings

Table 4. Option No. 2 ($R_y = 250$)

i	1	2	3	4	5
X_i / Y_i	5.5/1.5	5.5/2.1	5.5/3.4	5.5/4.2	5.5/4.9
x_i^* / y_i^*	4.28/4.01	9.64/12.67	18.84/40.07	29.14/76.55	38.07/116.67
p_i^x / p_i^y	0.52/0.47	0.49/0.5	0.43/0.56	0.33/0.66	0.21/0.78
F_x / F_y	10.87/10.24				

Calculated value $\lambda = 3.842$.

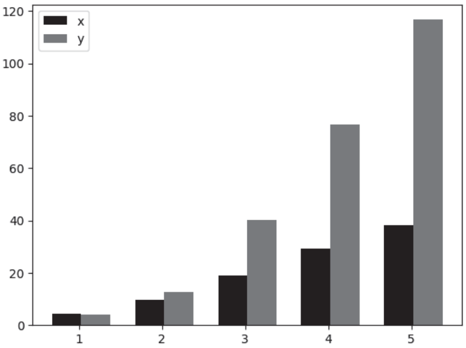

Fig. 7. Optimal solutions of players

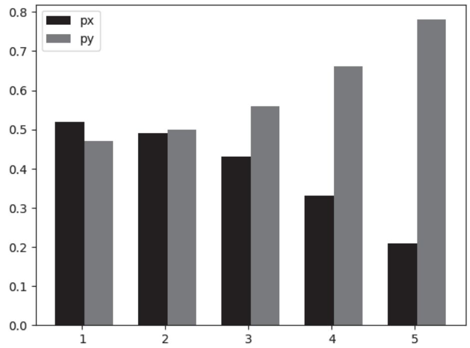

Fig. 8. Probabilities of players' winnings

The third case.

Table 5. Option No. 1 ($R_y = 350$)

i	1	2	3	4	5
X_i / Y_i	1.1/1.5	1.1/2.1	1.1/3.4	1.1/4.2	1.1/4.9
x_i^* / y_i^*	4.06/5.37	9.93/18.38	20.34/60.94	29.33/108.53	36.31/156.75
p_i^x / p_i^y	0.51/0.49	0.47/0.52	0.39/0.6	0.28/0.71	0.15/0.84
F_x / F_y	1.95/10.94				

Calculated value $\lambda = 0.967$.

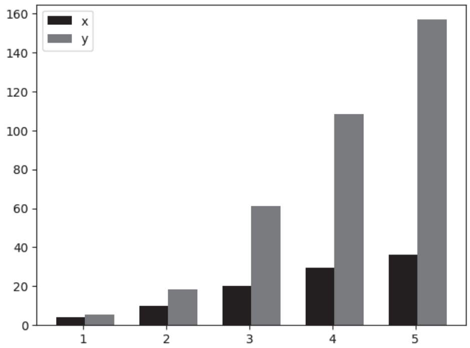

Fig. 9. Optimal solutions of players

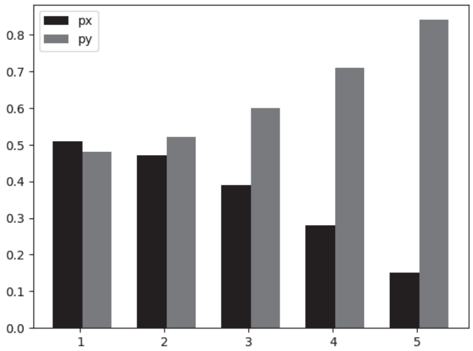

Fig. 10. Probabilities of players' winnings

Table 6. Option No. 2 ($R_y = 350$)

i	1	2	3	4	5
X_i / Y_i	5.5/1.5	5.5/2.1	5.5/3.4	5.5/4.2	5.5/4.9
x_i^* / y_i^*	4.77/6.34	10.42/19.4	19.5/58.78	29.19/108.67	36.1/156.78
p_i^x / p_i^y	0.51/0.48	0.47/0.52	0.38/0.61	0.28/0.71	0.15/0.84
F_x / F_y	9.83/10.97				

Calculated value $\lambda = 5.689$.

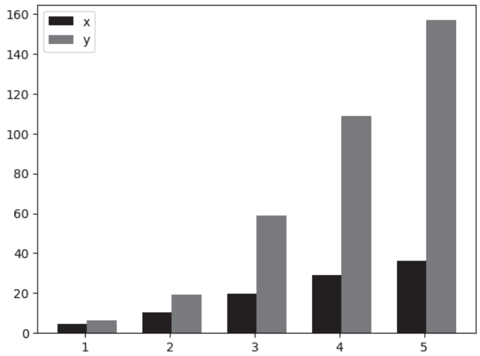

Fig. 11. Optimal solutions of players

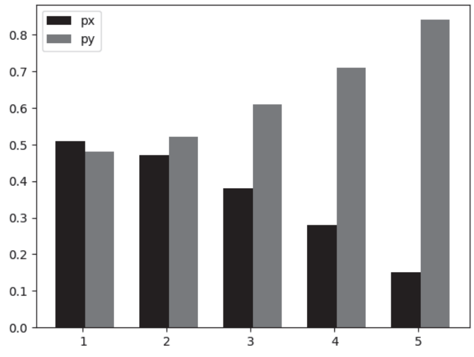

Fig. 12. Probabilities of players' winnings

Let us analyze the first case. From Table 1 it can be seen that the optimal values of the vector components x^* are greater than the corresponding vector components y^* at the first and second sites. At the same time, as follows from Fig. 1 and Fig. 2, despite the lack of advantage of the first player on the third court, the probability of his victory on the indicated court is almost equal to this value for the opponent. If the values of all the first player's sites increase fivefold, the overall picture presented in Table 2, Fig. 3 and Fig. 4 remains the same. Changes occur for the parameter λ.

Let's move on to the second case. The opponent increases his resources by 100. As can be seen from Table 3 and Fig. 5, the first player has a small advantage on the first site among the components of the optimal solution. A similar situation persists in the probability of winnings. However, as shown in Fig. 6, on the second court the probability of winning for the second player is slightly higher than that for the first. Increasing the value of each site fivefold, as follows from Table 4, Fig. 7 and Fig. 8, again does not change the situation as a whole, except for the value of the parameter λ. In general, the increase in the opponent's resources led to the loss of one site where the first player had previously won.

In the third case, we will again increase the opponent's resources by 100. As follows from Table 5, Fig. 9, Fig. 10 and Table 6, Fig. 11, Fig. 12, the components of the optimal vector of the first player are smaller than similar components of the second, but the probability of winning on the first and slightly smaller on the second platforms for the first player remains higher. Thus, the overall picture remains the same compared to the second case. It is clear that with a further increase in the opponent's resources, his winning probabilities will exceed those of the first player. Hence, the problem of the ratio of players' resources when the opponent's winning probabilities become larger on all platforms is of interest.

CONCLUSIONS

The case of a probabilistic model for two players is considered. A precise solution for the asymmetric Colonel Blotto game without the homogeneity constraint is found. The optimal allocation of players' resources is obtained in the form of a Nash equilibrium in pure strategies of the game.

A method for reducing the dimensionality of a system of equations obtained using the Lagrange multiplier method is proposed.

Of further interest is the study of solutions under conditions of incomplete information, that is, situations where the values of the coefficients are not precisely known.

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ЙМОВІРНІСНА МОДЕЛЬ ГРИ ПОЛКОВНИКА БЛОТТО БЕЗ ОБМЕЖЕНЬ СИМЕТРИЧНОСТІ ТА ОДНОРІДНОСТІ / С.А. Смирнов, І.М. Терещенко

Анотація. Розглянуто класичну гру полковника Блотто для двох гравців. Досліджено ймовірнісну модель вказаної задачі, причому на гру не накладаються обмеження симетричності та однорідності. З метою пошуку розв’язку знайдено спосіб пониження розмірності, оскільки одержана за допомогою методу множників Лагранжа система рівнянь має велику розмірність. Знайдене співвідношення між ресурсами обох гравців, що розподілені по майданчиках, дало змогу виділити параметр, який визначається співвідношенням множників Лагранжа з відповідних функцій для обох гравців. Для такого параметра знайдено інтервальне обмеження, яке він задовольняє, та для його пошуку сформульовано рівняння, яке розв’язується чисельно. Знайдене значення параметру дає можливість розрахувати окремі множники Лагранжа та отримати оптимальний розподіл ресурсів гравців у вигляді рівноваги Неша в чистих стратегіях гри. Досліджено приклад гри за суттєво відмінних умов для гравців.

Ключові слова: конфліктне протиборство, оптимальний розподіл ресурсів, гра двох осіб, гра полковника Блотто, ймовірнісна модель виграшу, рівновага Неша, ефективність використання ресурсів, 1-параметризація.